IMC Follow-up 2

Year 10/11 Pink Kangaroo

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Comments and suggestions to DrYuFromShanghai@QQ.com

Answers:

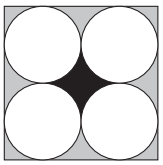
	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	
1	C	C	C		A	D	C	D	D	D	D	C	C	B	B	C	D	B	B	1
2	A	A	C		A	C	A	D	A	C	C	E	B	A	E	C	B	B	C	2
3	A	C	B		C	B	C	C	C	A	C	A	A	B	D	E	E	B	E	3
4	E	E	B		D	B	B	D	D	C	E	C	E	D	D	A	C	C	C	4
5	D	D	D		E	C	B	E	C	E	E	E	E	C	C	B	B	B	B	5
6	D	D	B		A	C	E	C	C	B	C	C	E	D	A	B	D	E	E	6
7	B	C	E		B	B	B	A	E	D	A	C	C	C	D	A	B	B	D	7
8	B	A	D		B	D	E	C	B	D	A	B	B	E	C	D	A	A	B	8
9	C	D	E		C	E	B	D	B	C	D	D	C	D	E	B	B	B	A	9
10	D	B	C		B	C	E	B	A	B	B	A	C	A	C	D	C	A	C	10
11	B	B	B		C	B	A	B	C	B	C	E	D	E	B	C	C	C	E	11
12	D	D	D		A	A	C	C	A	C	E	E	C	C	B	E	E	E	B	12
13	B	C	B		B	D	D	D	E	B	A	C	A	B	A	B	D	D	A	13
14	C	E	C		B	A	A	B	E	A	A	D	B	D	C	A	C	A	E	14
15	D	B	E		E	B	C	C	D	E	A	C	E	C	A	D	C	D	D	15
16	A	D	D		D	D	E	A	A	D	B	D	E	B	A	E	D	B	E	16
17	E	D	C		C	D	C	B	B	E	D	D	E	A	D	B	E	C	A	17
18	E	C	B		C	E	C	B	D	A	D	D	B	C	C	A	A	E	B	18
19	A	C	D		C	E	C	B	C	C	E	B	C	B	C	C	D	D	E	19
20	B	B	A		B	A	B	A	B	B	B	C	D	E	E	E	C	C	A	20
21	C	E	C		E	B	A	A	C	D	E	C	C	A	B	B	C	C	B	21
22	B	B	E		D	C	B	D	A	C	B	B	A	C	E	D	D	C	E	22
23	D	B	E		D	A	B	E	C	C	B	E	B	A	B	B	B	A	D	23
24	C	D	D		C	E	C	A	E	B	D	B	C	B	D	D	A	E	D	24
25	C	D	A		D	D	D	D	E	B	A	C	B	D	A	D	C	B	C	25

Comments and suggestions to DrYuFromShanghai@QQ.com

1. What is the value of $\dfrac{2 \times 0.24}{20 \times 2.4}$?

A 0.01 B 0.1 C 1 D 10 E 100

2. The figure shows a square with four circles of equal area, each touching two sides of the square and two other circles. What is the ratio between the area of the black region and the total area of the grey regions?

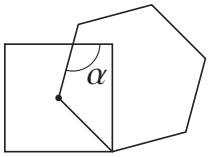


A 1 : 4 B 1 : 3 C 2 : 3 D 3 : 4 E π : 1

3. 232 and 111 are both three-digit palindromes as they read the same from left to right as they do right to left. What is the sum of the digits of the largest three-digit palindrome that is also a multiple of 6?

A 16 B 18 C 20 D 21 E 24

4. Tom draws a square. He adds a regular hexagon, one side of which joins the centre of the square to one of the vertices of the square, as shown. What is the size of angle α?

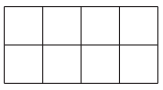


A 105° B 110° C 115° D 120° E 125°

5. Sadinie is asked to create a rectangular enclosure using 40 m of fencing so that the side-lengths, in metres, of the enclosure are all prime numbers. What is the maximum possible area of the enclosure?

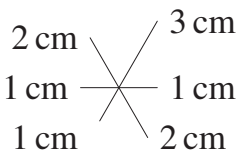
A 51 m^2 B 84 m^2 C 91 m^2 D 96 m^2 E 99 m^2

6. Lil writes one of the letters P, Q, R, S in each cell of a 2×4 table. She does this in such a way that, in each row and in each 2×2 square, all four letters appear. In how many ways can she do this?



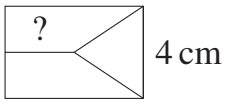
A 12 B 24 C 48 D 96 E 198

7. The diagram gives the lengths of six lines meeting at a point. Eric wishes to redraw this diagram without lifting his pencil from the paper. He can choose to start his drawing anywhere. What is the shortest distance he can draw to reproduce the figure?



A 14 cm B 15 cm C 16 cm D 17 cm E 18 cm

8. A rectangle is divided into three regions of equal area. One of the regions is an equilateral triangle with side-length 4 cm; and the other two are trapezia, as shown.



What is the length of the smaller of the parallel sides of the trapezia?

A $\sqrt{2}$ cm B $\sqrt{3}$ cm C $2\sqrt{2}$ cm D 3 cm E $2\sqrt{3}$ cm

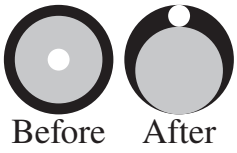
9. The ages of Jo, her daughter and her grandson are all even numbers. The product of their three ages is 2024. How old is Jo?

A 42 B 44 C 46 D 48 E 50

10. The sum of the digits of the positive integer N is three times the sum of the digits of $N + 1$. What is the smallest possible sum of the digits of N?

A 9 B 12 C 15 D 18 E 27

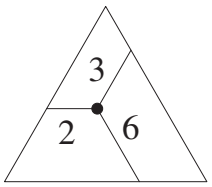
11. Polly has three circles cut from three pieces of coloured card. She originally places them on top of each other as shown. In this configuration the area of the visible black region is seven times the area of the white circle.



Polly moves the circles to a new position, as shown, with each pair of circles touching each other. What is the ratio between the areas of the visible black regions before and after?

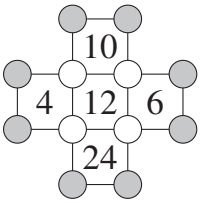
A 3 : 1 B 4 : 3 C 6 : 5 D 7 : 6 E 9 : 7

12. A point is chosen inside an equilateral triangle. From this point we draw three segments parallel to the sides, as shown. The lengths of the segments are 2 m, 3 m and 6 m. What is the perimeter of the triangle?



A 22 m B 27 m C 33 m D 39 m E 44 m

13. A number is written in each of the twelve circles shown. The number inside each square indicates the product of the numbers at its four vertices. What is the product of the numbers in the eight grey circles?



A 20 B 40 C 80 D 120 E 480

14. The sides, in cm, of two squares are integers. The difference between the areas of the two squares is 19 cm^2. What is the sum of the perimeters of the two squares?

A 38 cm B 60 cm C 64 cm D 72 cm E 76 cm

15. Molly has a set of cards numbered 1 to 12. She places eight of them at the vertices of an octagon so that the sum of every pair of numbers at opposite ends of an edge of the octagon is a multiple of 3.

Which numbers did Molly not place?

A 1, 5, 9 and 12 B 3, 5, 7 and 9 C 1, 2, 11 and 12
D 5, 6, 7 and 8 E 3, 6, 9 and 12

16. Peter always tells the truth or always lies on alternate days. One day, he made exactly four of the following five statements. Which one did he not make?

A I lied yesterday and I will lie tomorrow.
B I am telling the truth today and I will tell the truth tomorrow.
C 2024 is divisible by 11.
D Yesterday was Wednesday.
E Tomorrow will be Saturday.

17. Matthew rolled a normal die 24 times. All numbers from 1 to 6 came up at least once. The number 1 came up more times than any other number. Matthew added up all the numbers. The total he obtained was the largest one possible. What total did he obtain?

A 83 B 84 C 89 D 90 E 100

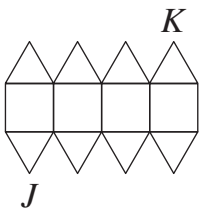
18. For some positive integer n, the prime factorisation of the number $n! = 1 \times 2 \times \ldots \times n$ is of the form shown below.

$$2^{\square} \times 3^{\square} \times 5^{\square} \times 7^{\square} \times 11^{\square} \times 13^{4} \times 17^{\square} \times \square^{\square} \times \square^{\square} \times 41 \times 43 \times 47$$

The primes are written in increasing order. Ink has covered some of the primes and some of the exponents. What is the exponent of 17?

A 1 B 2 C 3 D 4 E 5

19. Kevin makes a net using a combination of squares and equilateral triangles, as shown in the figure. The side length of each square and of each triangle is 1 cm. He folds the net up to form the surface of a polyhedron. What is the distance between the vertices J and K in this polyhedron?



A $\sqrt{5}$ cm B $(1+\sqrt{2})$ cm C $\frac{5}{2}$ cm

D $(1+\sqrt{3})$ cm E $2\sqrt{2}$ cm

20. Jill has some unit cubes which are all black, grey, or white. She uses 27 of them to build a $3 \times 3 \times 3$ cube. She wants the surface to be exactly one-third black, one-third grey, and one-third white. The smallest possible number of black cubes she can use is X and the largest possible number of black cubes she can use is Y. What is the value of $Y - X$?

A 1 B 3 C 6 D 7 E 9

21. Meera walked in the park. She walked half of the total time at a speed of 2 km/h. She then walked half of the total distance at a speed of 3 km/h. Finally, she completed the remainder of the walk at a speed of 4 km/h. For what fraction of the total time did she walk at a speed of 4 km/h?

A $\frac{1}{14}$ B $\frac{1}{12}$ C $\frac{1}{7}$ D $\frac{1}{5}$ E $\frac{1}{4}$

22. Given the integers from 1 to 25, Ajibola wants to remove a few and then split those that remain into two groups so that the products of the integers in each group are equal. Ajibola removes the smallest possible number of integers in order to achieve this. What is the sum of the numbers which Ajibola removes?

A 75 B 79 C 81 D 83 E 89

23. Twenty points are equally spaced around the circumference of a circle. Kevin draws all the possible chords that connect pairs of these points. How many of these chords are longer than the radius of the circle but shorter than its diameter?

A 90 B 100 C 120 D 140 E 160

24. There are n distinct lines in the plane. One of these lines intersects exactly 5 of the n lines, another of these intersects exactly 9 of the n lines, and yet another intersects exactly 11 of them. Which of the following is the smallest possible value of n?

A 12 B 13 C 14 D 25 E 28

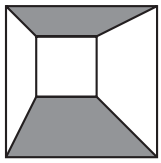
25. Suppose m and n are integers with $0 < m < n$. Let $P = (m, n)$, $Q = (n, m)$, and $O = (0, 0)$. For how many pairs of m and n will the area of triangle OPQ be equal to 2024?

A 4 B 6 C 8 D 10 E 12

1. Kangaroo Day this year falls on Thursday 16th March 2023. What day will it be in 2023 days' after this date?

A Sunday B Monday C Tuesday D Wednesday E Thursday

2. A large square of side-length 10 cm contains a smaller square of side-length 4 cm, as shown in the diagram. The corresponding sides of the two squares are parallel. What percentage of the area of the large square is shaded?

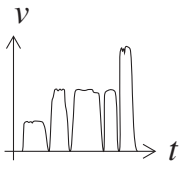


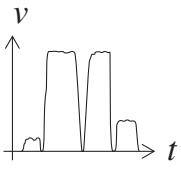
A 25% B 30% C 40% D 42% E 45%

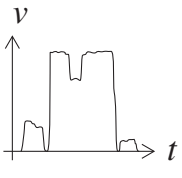
3. A wooden fence consists of a series of vertical planks, each joined to the next vertical plank by four horizontal planks. The first and last planks in the fence are vertical. Which of the following could be the total number of planks in the fence?

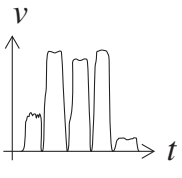
A 96 B 97 C 98 D 99 E 100

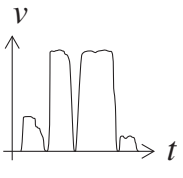
4. Mary had to run to catch the train, got off two stops later and then walked to school. Which of the following speed-time graphs would best represent her journey?

A 

B 

C 

D 

E 

5. Alec has won 49% of the 200 games of chess he has played. He would like to have won exactly 50% of his games. What is the smallest number of extra games he needs to play?

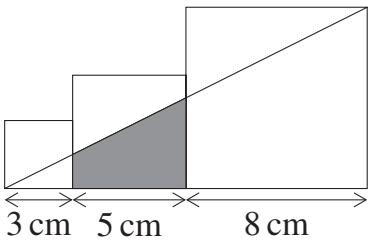
A 0 B 1 C 2 D 3 E 4

6. Lucy is trying to save water. She has reduced the time she spends in her shower by one quarter. Also, she has lowered the water pressure to reduce the rate the water comes out of the shower head by a quarter. By what fraction has Lucy reduced the total amount of water she uses when she showers?

A $\frac{3}{8}$ B $\frac{1}{16}$ C $\frac{5}{12}$ D $\frac{7}{16}$ E $\frac{9}{16}$

7. The diagram shows three squares of side-length 3 cm, 5 cm and 8 cm. What is the area, in cm^2, of the shaded trapezium?

A 13 B $\frac{55}{4}$ C $\frac{61}{4}$ D $\frac{65}{4}$ E $\frac{69}{4}$



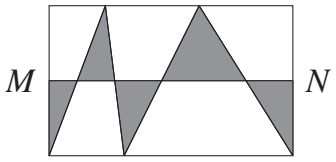
8. A wire of length 95 m is cut into three pieces such that the length of each piece is 50 % greater than the previous piece. What is the length of the largest piece?

A 36 m B 42 m C 45 m D 46 m E 48 m

9. The ages of a family of five sum to 80. The two youngest are 6 and 8. What was the sum of the ages of the family seven years ago?

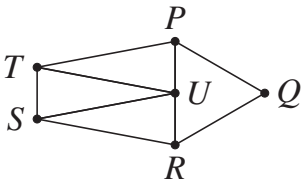
A 35 B 36 C 45 D 46 E 66

10. Points M and N are the midpoints of two sides of the rectangle, shown in the diagram. What fraction of the rectangle's area is shaded?



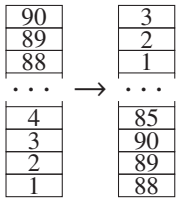
A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$

11. The Pentagon $PQRST$ is divided into four triangles with equal perimeters. The triangle PQR is equilateral. PTU, SUT and RSU are congruent isosceles triangles. What is the ratio of the perimeter of the pentagon $PQRST$ to the perimeter of the triangle PQR?



A 2 : 1 B 3 : 2 C 4 : 3 D 5 : 3 E 5 : 2

12. On the table there is a tower made of blocks numbered from 1 to 90, as shown on the left of the diagram. Yett takes blocks from the top of the tower, three at a time, to build a new tower, as shown on the right of the diagram. How many blocks will be between blocks 39 and 40 when he has finished building the new tower?

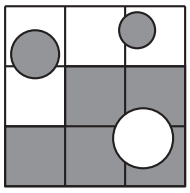


A 0 B 1 C 2 D 3 E 4

13. We will call a two-digit number **power-less** if neither of its digits can be written as an integer to a power greater than 1. For example, 53 is power-less, but 54 is *not* power-less since $4 = 2^2$. Which of the following is a common divisor of the smallest and the largest power-less numbers?

A 3 B 5 C 7 D 11 E 13

14. A square of side-length 30 cm is divided into nine smaller identical squares. The large square contains three circles with radii 5 cm (bottom right), 4 cm (top left) and 3 cm (top right), as shown. What is the total area of the shaded part?

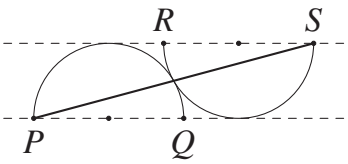


A $400\,\text{cm}^2$ B $500\,\text{cm}^2$ C $(400+50\pi)\text{cm}^2$
D $(500-25\pi)\text{cm}^2$ E $(500+25\pi)\text{cm}^2$

15. Jenny calculates the average of five different prime numbers. Her answer is an integer. What is the smallest possible integer she could have obtained?

A 2 B 5 C 6 D 12 E 30

16. The figure shows two touching semicircles of radius 1, with parallel diameters PQ and RS. What is the square of the distance PS?



A 16 B $8+4\sqrt{3}$ C 12 D 9
E $5+2\sqrt{3}$

17. Ireena is extending a sequence of numbers with the following rule. The next term in the sequence is the smallest non-negative integer that is different from each of the four preceding terms. She then repeats this process over and over again. For instance, if Ireena was to start with the sequence 7, 3, 1, 8 then the fifth and sixth terms of the sequence would be 0 and 2 respectively.

Ireena starts with the sequence

2, 0, 2, 3.

What is the 2023rd number in this sequence?

A 0 B 1 C 2 D 3 E 4

18. A group of students took a test which consisted of 3 questions. We know that 90 % answered the first question correctly, 80 % answered the second question correctly and 70 % answered the third question correctly. What is the smallest possible percentage of students who answered all three questions correctly?

A 30 % B 35 % C 40 % D 50 % E 70 %

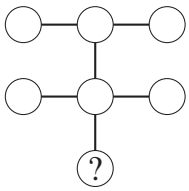
19. A rectangle with vertices $(0, 0)$, $(100, 0)$, $(100, 50)$ and $(0, 50)$ has a circle with centre $(75, 30)$ and radius 10 cut out of it. What is the slope of the line through the point $(75, 30)$ which divides the remaining area of the rectangle into two shapes of equal area?

A $\frac{1}{5}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{5}$ E $\frac{2}{3}$

20. Eva chooses a three-digit positive number and from it she subtracts the sum of its three digits. She finds that the answer is a three-digit number in which all three digits are the same. How many different starting numbers could Eva have chosen?

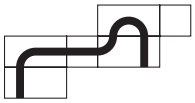
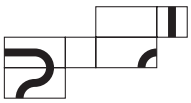
A 2 B 5 C 10 D 20 E 30

21. Seven different single-digit numbers are written in the circles of the diagram shown with one number in each circle. The product of the three numbers in each of the three lines of three numbers is the same. Which number is written in the circle containing the question mark?



A 2 B 3 C 4 D 6 E 8

22. Lancelot has drawn a closed path on a cuboid and unfolded it into a net. Which of the nets shown could *not* be the net of Lancelot's cuboid?

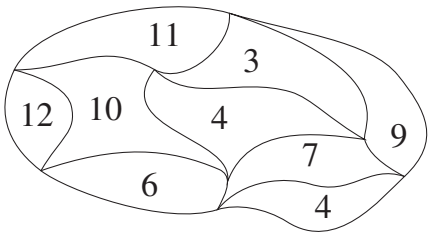
A  B  C  D  E 

23. In how many different ways can the word **BANANA** be read from the following table by moving from one cell to another cell with which it shares an edge? Cells may be visited more than once.

B	A	N
A	N	A
N	A	N

A 14 B 28 C 56 D 84 E 112

24. The diagram shows a map of a park. The park is divided into regions. The number inside each region gives its perimeter, in km. What is the outer perimeter of the park?

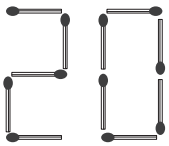


A 22 km B 26 km C 28 km D 32 km E 34 km

25. Vumos wants to write the integers 1 to 9 in the nine boxes shown so that the sum of the integers in any three adjacent boxes is a multiple of 3. In how many ways can he do this?

A $6 \times 6 \times 6 \times 6$ B $6 \times 6 \times 6$
C $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ D $6 \times 5 \times 4 \times 3 \times 2 \times 1$
E $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

1. Carolina has a box of 30 matches. She begins to make the number 2022 using matchsticks. The diagram shows the first two digits.
How many matchsticks will be left in the box when she has finished?

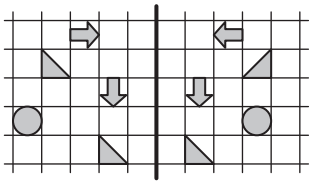


A 20 B 19 C 10 D 9 E 5

2. A square has the same perimeter as an equilateral triangle whose sides all have length 12 cm.
What is the length, in cm, of the sides of the square?

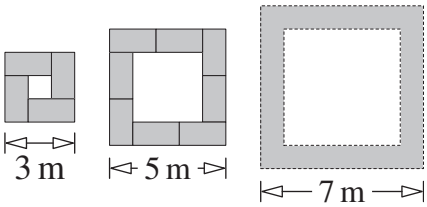
A 9 B 12 C 16 D 24 E 36

3. Some shapes are drawn on a piece of paper. The teacher folds the left-hand side of the paper over the central bold line. How many of the shapes on the left-hand side will fit exactly on top of a shape on the right-hand side?



A 1 B 2 C 3 D 4 E 5

4. Katrin arranges tables measuring 2 m by 1 m according to the number of participants in a meeting. The diagrams show the plan view for a small, a medium and a large meeting.
How many tables are needed for a large meeting?



A 10 B 11 C 12 D 14 E 16

5. On Nadya's smartphone, the diagram shows how much time she spent last week on four of her apps. This week she halved the time spent on two of these apps, but spent the same amount of time as the previous week on the other two apps.
Which of the following could be the diagram for this week?

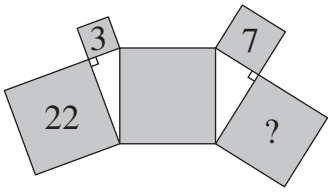
A  B  C  D  E 

6. There were five candidates in the school election. After 90% of the votes had been counted, the preliminary results were as shown on the right. How many students still had a chance of winning the election?

Henry	India	Jenny	Ken	Lena
14	11	10	8	2

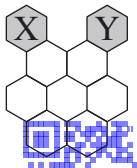
A 1 B 2 C 3 D 4 E 5

7. Five squares and two right-angled triangles are positioned as shown. The areas of three squares are 3 m^2, 7 m^2 and 22 m^2 as shown.
What is the area, in m^2, of the square with the question mark?



A 18 B 19 C 20 D 21 E 22

8. A ladybird aims to travel from hexagon X to hexagon Y, passing through each of the seven unshaded hexagons once and only once. She can move from one hexagon to another only through a common edge. How many different routes could she take?

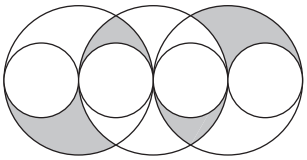


A 2 B 3 C 4 D 5 E 6

9. Adam laid 2022 tiles in a long line. Beata removed every sixth tile. Carla then removed every fifth tile. Doris then removed every fourth tile. Lastly, Eric removed all of the remaining tiles.
How many tiles did Eric remove?

A 0 B 337 C 674 D 1011 E 1348

10. The centres of the seven circles shown all lie on the same line. The four smaller circles have radius 1 cm. The circles touch, as shown.
What is the total area of the shaded regions?

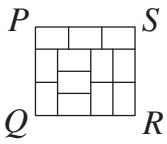


A $\pi\,\text{cm}^2$ B $2\pi\,\text{cm}^2$ C $3\pi\,\text{cm}^2$ D $4\pi\,\text{cm}^2$ E $5\pi\,\text{cm}^2$

11. Gran's first grandchild guessed that Gran was 75, the second 78 and the third 81. It turned out that one of them was mistaken by 1 year, another one by 2 years and the other by 4 years. What is Gran's age?

A 76 B 77 C 78 D 79
E impossible to determine

12. Twelve congruent rectangles are placed together to make a rectangle $PQRS$ as shown. What is the ratio $PQ : QR$?

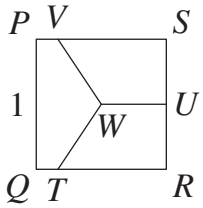


A 2 : 3 B 3 : 4 C 5 : 6 D 7 : 8 E 8 : 9

13. A rabbit and a hedgehog participated in a running race on a 550 m long circular track, both starting and finishing at the same point. The rabbit ran clockwise at a speed of 10 m/s and the hedgehog ran anticlockwise at a speed of 1 m/s. When they met, the rabbit continued as before, but the hedgehog turned round and ran clockwise. How many seconds after the rabbit did the hedgehog reach the finish?

A 25 B 45 C 50 D 55 E 100

14. The diagram shows a square $PQRS$ of side-length 1. W is the centre of the square and U is the midpoint of RS. Line segments TW, UW and VW split the square into three regions of equal area. What is the length of SV?

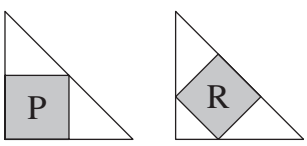


A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

15. Eight teams participated in a football tournament, and each team played exactly once against each other team. If a match was drawn then both teams received 1 point; if not then the winner of the match was awarded 3 points and the loser received no points. At the end of the tournament the total number of points gained by all the teams was 61.
What is the maximum number of points that the tournament's winning team could have obtained?

A 16 B 17 C 18 D 19 E 21

16. Two congruent isosceles right-angled triangles each have squares inscribed in them as shown. The square P has an area of 45 cm^2.
What is the area, in cm^2, of the square R?



A 40 B 42 C 45 D 48 E 50

17. Veronica put on five rings: one on her little finger, one on her middle finger and three on her ring finger. In how many different orders can she take them all off one by one?

A 16 B 20 C 24 D 30 E 45

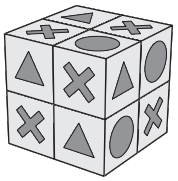
18. A certain city has two types of people: the 'positives', who only ask questions for which the correct answer is "yes" and the 'negatives' who only ask questions for which the correct answer is "no". When Mo and Bo met Jo, Mo asked, "Are Bo and I both negative?" What can be deduced about Mo and Bo?

A Both positive B Both negative C Mo negative, Bo positive
D Mo positive, Bo negative E impossible to determine

19. A group of pirates (raiders, sailors and cabin boys) divided 200 gold and 600 silver coins between them. Each raider received 5 gold and 10 silver coins. Each sailor received 3 gold and 8 silver coins. Each cabin boy received 1 gold and 6 silver coins. How many pirates were there altogether?

A 50 B 60 C 72 D 80 E 90

20. Cuthbert is going to make a cube with each face divided into four squares. Each square must have one shape drawn on it; either a cross, a triangle or a circle. Squares that share an edge must have different shapes on them. One possible cube is shown in the diagram. Which of the following combinations of crosses and triangles is possible on such a cube (with the other shapes being circles)?

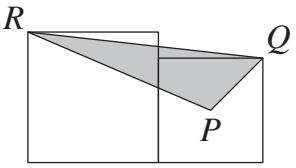


A 6 crosses, 8 triangles B 7 crosses, 8 triangles C 5 crosses, 8 triangles
D 7 crosses, 7 triangles E none of these are possible

21. A grocer has twelve weights, weighing 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 kilograms respectively. He splits them into three groups of four weights each. The total weights of the first and second groups are 41 kg and 26 kg respectively. Which of the following weights is in the same group as the 9 kg weight?

A 3 kg B 5 kg C 7 kg D 8 kg E 10 kg

22. The bases of the two touching squares shown lie on the same straight line. The lengths of the diagonals of the larger square and the smaller square are 10 cm and 8 cm respectively. P is the centre of the smaller square. What is the area, in cm^2, of the shaded triangle PQR?



A 18 B 20 C 22 D 24 E 26

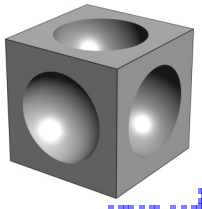
23. The product of the digits of the positive integer N is 20.
One of the following could *not* be the product of the digits of $N + 1$. Which is it?

A 24 B 25 C 30 D 35 E 40

24. The lengths of the sides of pentagon $ABCDE$ are as follows: $AB = 16$ cm, $BC = 14$ cm, $CD = 17$ cm, $DE = 13$ cm, $AE = 14$ cm. Five circles with centres at the points A, B, C, D, E are drawn so that each circle touches both of its immediate neighbours. Which point is the centre of the largest circle?

A B C D E

25. The cube shown has sides of length 2 units. Holes in the shape of a hemisphere are carved into each face of the cube. The six hemispheres are identical and their centres are at the centres of the faces of the cube. The holes are just large enough to touch the hole on each neighbouring face. What is the diameter of each hole?

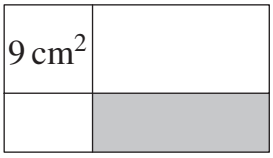


A 1 B $\sqrt{2}$ C $2-\sqrt{2}$ D $3-\sqrt{2}$ E $3-\sqrt{3}$

1. The mean age of the members of a jazz band is 21. The saxophonist, singer and trumpeter are 19, 20 and 21 years old respectively. The other three musicians are all the same age. How old are they?

A 21 B 22 C 23 D 24 E 26

2. A rectangle with perimeter 30 cm is divided by two lines, forming a square of area 9 cm^2, as shown in the figure.
What is the perimeter of the shaded rectangle?



A 14 cm B 16 cm C 18 cm D 21 cm E 24 cm

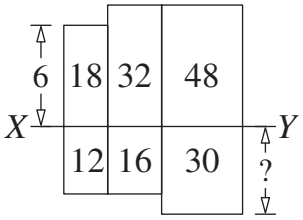
3. The number x has the following property: subtracting $\frac{1}{10}$ from x gives the same result as multiplying x by $\frac{1}{10}$. What is the number x?

A $\frac{1}{100}$ B $\frac{1}{11}$ C $\frac{1}{10}$ D $\frac{11}{100}$ E $\frac{1}{9}$

4. Six congruent rhombuses, each of area 5 cm^2, form a star. The tips of the star are joined to draw a regular hexagon, as shown. What is the area of the hexagon?

A 36 cm^2 B 40 cm^2 C 45 cm^2 D 48 cm^2 E 60 cm^2

5. Six rectangles are arranged as shown. The number inside each rectangle gives the area, in cm^2, of that rectangle. The rectangle on the top left has height 6 cm.
What is the height of the bottom right rectangle?

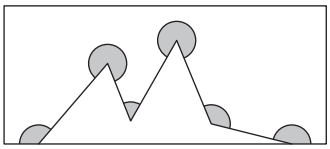


A 4 cm B 5 cm C 6 cm D 7.5 cm E 10 cm

6. How many five-digit positive integers have the product of their digits equal to 1000?

A 10 B 20 C 28 D 32 E 40

7. Five line segments are drawn inside a rectangle as shown.
What is the sum of the six marked angles?

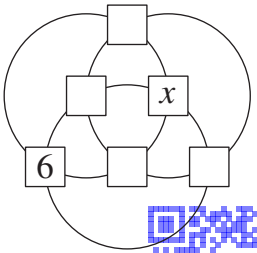


A 360° B 720° C 900° D 1080° E 1120°

8. At half-time in a handball match, the home team was losing 9–14 to the visiting team. However, in the second half, the home team scored twice as many goals as the visitors and won by one goal.
What was the full-time score?

A 20–19 B 21–20 C 22–21 D 23–22 E 24–23

9. The numbers from 1 to 6 are to be placed at the intersections of three circles, one number in each of the six squares. The number 6 is already placed.
Which number must replace x, so that the sum of the four numbers on each circle is the same?



A 1 B 2 C 3 D 4 E 5

10. Ahmad walks up a flight of eight steps, going up either one or two steps at a time. There is a hole on the sixth step, so he cannot use this step. In how many different ways can Ahmad reach the top step?

A 6 B 7 C 8 D 9 E 10

11. There were five teams entered in a competition. Each team consisted of either only boys or only girls. The number of team members was 9, 15, 17, 19 and 21. After one team of girls had been knocked out of the competition, the number of girls still competing was three times the number of boys. How many girls were in the team that was eliminated?

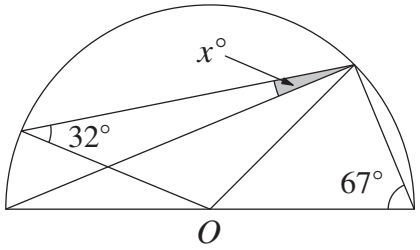
A 9 B 15 C 17 D 19 E 21

12. Tom had ten sparklers of the same size. Each sparkler took 2 minutes to burn down completely. He lit them one at a time, starting each one when the previous one had one tenth of the time left to burn. How long did it take for all ten sparklers to burn down?

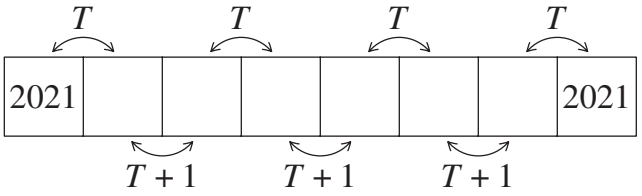
A 18 minutes and 20 seconds
B 18 minutes and 12 seconds
C 18 minutes
D 17 minutes
E 16 minutes and 40 seconds

13. The diagram shows a semicircle with centre O. Two of the angles are given. What is the value of x?

A 9 B 11 C 16 D 17.5 E 18



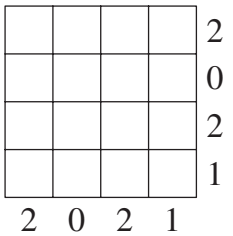
14. Each box in the strip shown is to contain one number. The first box and the eighth box each contain 2021. Numbers in adjacent boxes have sum T or $T+1$ as shown. What is the value of T?



A 4041 B 4042 C 4043 D 4044 E 4045

15. In the 4×4 grid some cells must be painted black. The numbers to the right of the grid and those below the grid show how many cells in that row or column must be black.
In how many ways can this grid be painted?

A 1 B 2 C 3
D 5 E more than 5



16. Five girls ran a race. Fiona started first, followed by Gertrude, then Hannah, then India and lastly Janice. Whenever a girl overtook another girl, she was awarded a point. India was first to finish, then Gertrude, then Fiona, then Janice and lastly Hannah.
What is the lowest total number of points that could have been awarded?

A 9 B 8 C 7 D 6 E 5

17. The number 2021 has a remainder of 5 when divided by 6, by 7, by 8, or by 9.
How many positive integers are there, smaller than 2021, that have this property?

A 4 B 3 C 2 D 1 E none

18. Tatiana's teacher drew a 3×3 grid on the board, with zero in each cell. The students then took turns to pick a 2×2 square of four adjacent cells, and to add 1 to each of the numbers in the four cells. After a while, the grid looked like the diagram on the right (some of the numbers in the cells have been rubbed out.)
What number should be in the cell with the question mark?

0	0	0
0	0	0
0	0	0

	18	
	47	
13		?

A 9 B 16 C 21 D 29 E 34

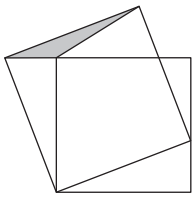
19. Three boys played a "Word" game in which they each wrote down ten words. For each word a boy wrote, he scored three points if neither of the other boys had the same word; he scored one point if only one of the other boys had the same word. No points were awarded for words which all three boys had. When they added up their scores, they found that they each had different scores. Sam had the smallest score (19 points), and James scored the most. How many points did James score?

A 20 B 21 C 23 D 24 E 25

20. Let N be the smallest positive integer such that the sum of its digits is 2021.
What is the sum of the digits of $N + 2021$?

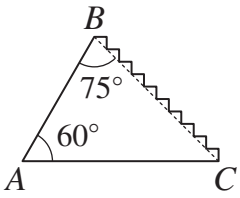
A 10 B 12 C 19 D 28 E 2021

21. The smaller square in the picture has area 16 and the grey triangle has area 1.
What is the area of the larger square?



A 17 B 18 C 19 D 20 E 21

22. A caterpillar crawled up a smooth slope from A to B, and crept down the stairs from B to C. What is the ratio of the distance the caterpillar travelled from B to C to the distance it travelled from A to B?



A 1:1 B 2:1 C 3:1 D $\sqrt{2}:1$ E $\sqrt{3}:1$

23. A total of 2021 balls are arranged in a row and are numbered from 1 to 2021. Each ball is coloured in one of four colours: green, red, yellow or blue. Among any five consecutive balls there is exactly one red, one yellow and one blue ball. After any red ball the next ball is yellow. The balls numbered 2 and 20 are both green. What colour is the ball numbered 2021?

A Green B Red C Yellow D Blue
E It is impossible to determine

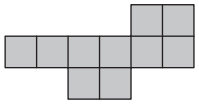
24. Each of the numbers m and n is the square of an integer. The difference $m - n$ is a prime number.
Which of the following could be n?

A 100 B 144 C 256 D 900 E 10000

25. Christina has eight coins whose weights in grams are different positive integers. When Christina puts any two coins in one pan of her balance scales and any two in the other pan of the balance scales, the side containing the heaviest of those four coins is always the heavier side.
What is the smallest possible weight of the heaviest of the eight coins?

A 8 B 12 C 34 D 55 E 256

1. The diagram shows a shape made from ten squares of side-length 1 cm, joined edge to edge.



What is the length of its perimeter, in centimetres?

A 14 B 18 C 30 D 32 E 40

2. When the answers to the following calculations are put in order from smallest to largest, which will be in the middle?

A $1 + 23456$ B $12 + 3456$ C $123 + 456$ D $1234 + 56$ E $12345 + 6$

3. In the calculations shown, each letter stands for a digit. They are used to make some two-digit numbers. The two numbers on the left have a total of 79.

What is the total of the four numbers on the right?

A 79 B 158 C 869 D 1418 E 7979

$$\begin{array}{r} JK \\ +\,LM \\ \hline 79 \end{array} \qquad \begin{array}{r} JM \\ +\,LM \\ +\,JK \\ +\,LK \\ \hline ? \end{array}$$

4. The sum of four consecutive integers is 2. What is the least of these integers?

A -3 B -2 C -1 D 0 E 1

5. The years 2020 and 1717 both consist of a repeated two-digit number.

How many years after 2020 will it be until the next year which has this property?

A 20 B 101 C 120 D 121 E 202

6. Mary had ten pieces of paper. Some of them were squares, and the rest were triangles. She cut three squares diagonally from corner to corner. She then found that the total number of vertices of the 13 pieces of paper was 42.

How many triangles did she have before making the cuts?

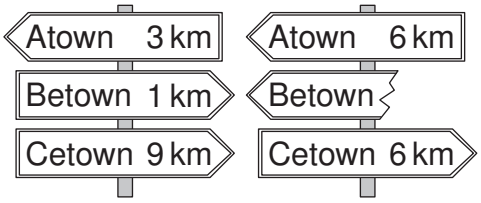
A 8 B 7 C 6 D 5 E 4

7. The positive integers a, b, c, d satisfy the equation $ab = 2cd$.

Which of the following numbers could not be the value of the product $abcd$?

A 50 B 100 C 200 D 450 E 800

8. The shortest path from Atown to Cetown runs through Betown. Two of the signposts that can be seen on this path are shown, but one of them is broken and a number missing.



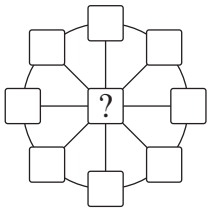
What distance was written on the broken sign?

A 2 km B 3 km C 4 km D 5 km E 6 km

9. An isosceles triangle has a side of length 20 cm. Of the remaining two side-lengths, one is equal to two-fifths of the other. What is the length of the perimeter of this triangle?

A 36 cm B 48 cm C 60 cm D 90 cm E 120 cm

10. Freda wants to write a number in each of the nine cells of this figure so that the sum of the three numbers on each diameter is 13 and the sum of the eight numbers on the circumference is 40.



What number must be written in the central cell?

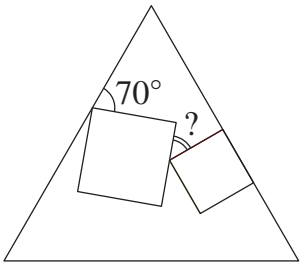
A 3 B 5 C 8 D 10 E 12

11. Masha put a multiplication sign between the second and third digits of the number 2020 and noted that the resulting product 20×20 was a square number.

How many integers between 2010 and 2099 (including 2020) have the same property?

A 1 B 2 C 3 D 4 E 5

12. Two squares of different sizes are drawn inside an equilateral triangle. One side of one of these squares lies on one of the sides of the triangle as shown. What is the size of the angle marked by the question mark?



A 25° B 30° C 35° D 45° E 50°

13. Luca began a 520 km trip by car with 14 litres of fuel in the car tank. His car consumes 1 litre of fuel per 10 km. After driving 55 km, he saw a road sign showing the distances from that point to five petrol stations ahead on the road. These distances are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the car's fuel tank is 40 litres and Luca wants to stop just once to fill the tank.

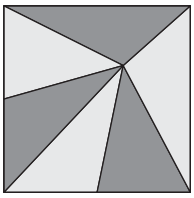
How far is the petrol station that he should stop at?

A 35 km B 45 km C 55 km D 75 km E 95 km

14. The numbers x and y satisfy the equation $17x + 51y = 102$. What is the value of $9x + 27y$?

A 54 B 36 C 34 D 18
E The value is undetermined.

15. A vertical stained glass square window of area 81 cm^2 is made out of six triangles of equal area (see figure). A fly is sitting on the exact spot where the six triangles meet. How far from the bottom of the window is the fly sitting?



A 3 cm B 5 cm C 5.5 cm D 6 cm E 7.5 cm

16. The digits from 1 to 9 are randomly arranged to make a 9-digit number.

What is the probability that the resulting number is divisible by 18?

A $\frac{1}{3}$ B $\frac{4}{9}$ C $\frac{1}{2}$ D $\frac{5}{9}$ E $\frac{3}{4}$

17. A hare and a tortoise competed in a 5 km race along a straight line, going due North. The hare is five times as fast as the tortoise. The hare mistakenly started running due East. After a while he realised his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What was the distance between the hare's turning point and the finish point?

A 11 km B 12 km C 13 km D 14 km E 15 km

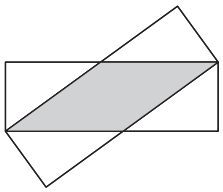
18. There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these shapes are large and the rest are small. We know that

1. If the shape is large, it's a square;
2. If the shape is blue, it's a triangle.

Which of the statements A–E must be true?

A All red figures are squares. B All squares are large. C All small figures are blue.
D All triangles are blue. E All blue figures are small.

19. Two identical rectangles with sides of length 3 cm and 9 cm are overlapping as in the diagram. What is the area of the overlap of the two rectangles?



A 12 cm^2 B 13.5 cm^2 C 14 cm^2 D 15 cm^2 E 16 cm^2

20. Kanga labelled the vertices of a square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums equalled 7, 8, 9 and 10. What is the sum for the fifth face?

A 11 B 12 C 13 D 14 E 15

21. A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that can have exactly one face painted?

A 27 B 28 C 32 D 34 E 40

22. In each of the cells, a number is to be written so that the sum of the 4 numbers in each row and in each column are the same.

What number must be written in the shaded cell?

1		6	3
	2	2	8
	7		4
		7	

A 5 B 6 C 7 D 8 E 9

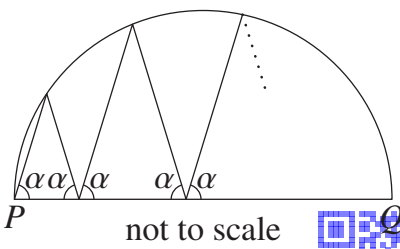
23. Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

A Alice
B Belle
C Cathy
D Either Alice or Belle could have lost the second game.
E Either Belle or Cathy could have lost the second game.

24. Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of these eight integers?

A 9 B 10 C 11 D 12 E 13

25. A zig-zag line starts at the point P, at one end of the diameter PQ of a circle. Each of the angles between the zig-zag line and the diameter PQ is equal to α as shown. After four peaks, the zig-zag line ends at the point Q. What is the size of angle α?



not to scale

A 60° B 72° C 75° D 80° E 86°

1. What is the value of $20 \times 19 + 20 + 19$?

A 389 B 399 C 409 D 419 E 429

2. A model train takes exactly 1 minute and 11 seconds for one complete circuit of its track. How long does it take for six complete circuits?

A 6 minutes and 56 seconds B 7 minutes and 6 seconds C 7 minutes and 16 seconds
D 7 minutes and 26 seconds E 7 minutes and 36 seconds

3. A barber wants to write the word SHAVE on a board behind the client's seat in such a way that a client looking in the mirror reads the word correctly. Which of the following should the barber write on the board?

A SHAVE B SHAVƎ C ƎVAHS D EVAHS E ƎVAHS

4. How many different totals can be obtained by rolling three standard dice and adding up the scores?

A 14 B 15 C 16 D 17 E 18

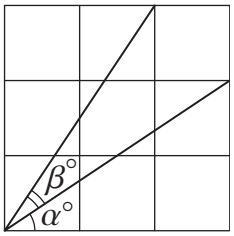
5. A park has five gates. In how many ways can Monica choose a gate to enter the park and a different gate to leave the park?

A 25 B 20 C 16 D 15 E 10

6. Pedro is asked to find three kangaroos whose weights are all whole numbers of kilograms and whose total weight is 97 kg. What is the largest possible weight of the lightest of the kangaroos Pedro could find?

A 1 kg B 30 kg C 31 kg D 32 kg E 33 kg

7. Two angles are marked on the 3×3 grid of squares.



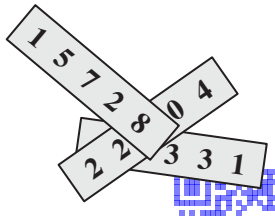
Which of the following statements about the angles is correct?

A $\alpha = \beta$ B $2\alpha + \beta = 90$ C $\alpha + \beta = 60$ D $2\beta + \alpha = 90$
E $\alpha + \beta = 45$

8. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

A  B  C  D  E 

9. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. What are the covered digits?



A 0, 2 and 2 B 1, 2 and 9 C 2, 4 and 9 D 2, 7 and 8
E 5, 7 and 8