1 Differentiate with respect to x

a e^x b $3e^x$ c $\ln x$ d $\frac{1}{2}\ln x$

2 Differentiate with respect to t

a $7 - 2e^t$ b $3t^2 + \ln t$ c $e^t + t^5$ d $t^{\frac{3}{2}} + 2e^t$

e $2\ln t + \sqrt{t}$ f $2.5e^t - 3.5\ln t$ g $\frac{1}{t} + 8\ln t$ h $7t^2 - 2t + 4e^t$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y = 4x^3 + e^x$ b $y = 7e^x - 5x^2 + 3x$ c $y = \ln x + x^{\frac{5}{2}}$

d $y = 5e^x + 6\ln x$ e $y = \frac{3}{x} + 3\ln x$ f $y = 4\sqrt{x} + \frac{1}{4}\ln x$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 3x + e^x$, $x = 0$ b $f(x) = \ln x - x^2$, $x = 4$

c $f(x) = x^{\frac{1}{2}} + 2\ln x$, $x = 9$ d $f(x) = 5e^x + \frac{1}{x^2}$, $x = -\frac{1}{2}$

5 Find, in each case, any values of x for which $\frac{dy}{dx} = 0$.

a $y = 5\ln x - 8x$ b $y = 2.4e^x - 3.6x$ c $y = 3x^2 - 14x + 4\ln x$

6 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x) = 2e^x - 3x$, $f'(x) = 7$ b $f(x) = 15x + \ln x$, $f'(x) = 23$

c $f(x) = \frac{x^2}{8} - 2x + \ln x$, $f'(x) = -1$ d $f(x) = 30\ln x - x^2$, $f'(x) = 4$

7 Find the coordinates and the nature of any stationary points on each of the following curves.

a $y = e^x - 2x$ b $y = \ln x - 10x$ c $y = 2\ln x - \sqrt{x}$

d $y = 4x - 5e^x$ e $y = 7 + 2x - 4\ln x$ f $y = x^2 - 26x + 72\ln x$

8 Given that $y = x + ke^x$, where k is a constant, show that

$$(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

9 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate

a $y = e^x$, $x = 2$ b $y = \ln x$, $x = 3$ c $y = 0.8x - 2e^x$, $x = 0$

d $y = 5\ln x + \frac{4}{x}$, $x = 1$ e $y = x^{\frac{1}{3}} - 3e^x$, $x = 1$ f $y = \ln x - \sqrt{x}$, $x = 9$

10 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate

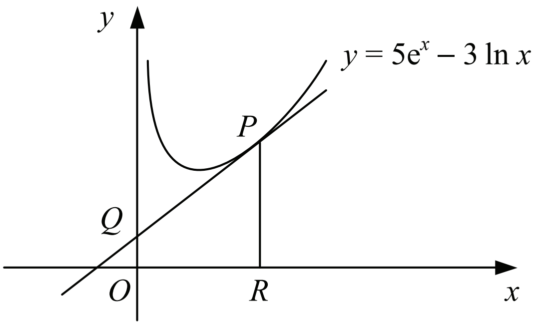
a $y = \ln x$, $x = e$ b $y = 4 + 3e^x$, $x = 0$ c $y = 10 + \ln x$, $x = 3$

d $y = 3\ln x - 2x$, $x = 1$ e $y = x^2 + 8\ln x$, $x = 1$ f $y = \frac{1}{10}x - \frac{3}{10}e^x - 1$, $x = 0$

1 **a** Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point where this normal crosses the x-axis.

2



The diagram shows the curve with equation $y = 5e^x - 3\ln x$ and the tangent to the curve at the point P with x-coordinate 1.

a Show that the tangent at P has equation $y = (5e - 3)x + 3$.

The tangent at P meets the y-axis at Q.
The line through P parallel to the y-axis meets the x-axis at R.

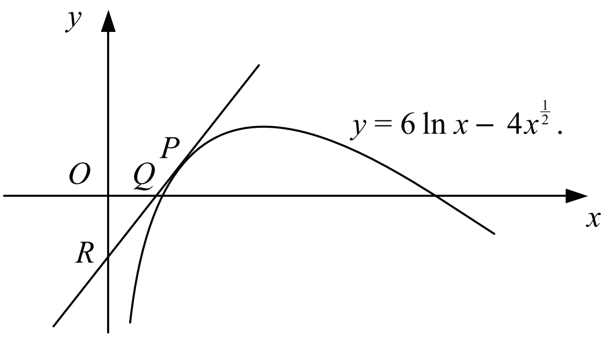
b Find the area of trapezium $ORPQ$, giving your answer in terms of e.

3 A curve has equation $y = 3x - \frac{1}{2}e^x$.

a Find the coordinates of the stationary point on the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

4



The diagram shows the curve $y = 6\ln x - 4x^{\frac{1}{2}}$. The x-coordinate of the point P on the curve is 4.

The tangent to the curve at P meets the x-axis at Q and the y-axis at R.

a Find an equation for the tangent to the curve at P.

b Hence, show that the area of triangle OQR is $(10 - 12\ln 2)^2$.

5 The curve with equation $y = 2x - 2 - \ln x$ passes through the point A (1, 0). The tangent to the curve at A crosses the y-axis at B and the normal to the curve at A crosses the y-axis at C.

a Find an equation for the tangent to the curve at A.

b Show that the mid-point of BC is the origin.

The curve has a minimum point at D.

c Show that the y-coordinate of D is $\ln 2 - 1$.

6 **a** Sketch the curve with equation $y = e^x + k$, where k is a positive constant.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.

Given that the tangent passes through the x-axis at the point $(-1, 0)$,

c show that $k = 2e^2$.

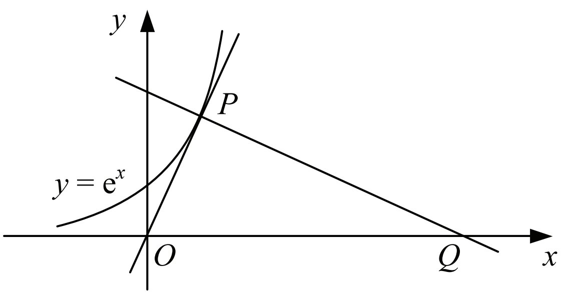
7 A curve has equation $y = 3x^2 - 2 \ln x$, $x > 0$.

The gradient of the curve at the point P on the curve is -1.

a Find the x-coordinate of P.

b Find an equation for the tangent to the curve at the point on the curve where $x = 1$.

8



The diagram shows the curve with equation $y = e^x$ which passes through the point $P\ (p, e^p)$. Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x-axis at Q,

a show that $p = 1$,

b show that the area of triangle OPQ, where O is the origin, is $\frac{1}{2}e(1 + e^2)$.

9 The curve with equation $y = 4 - e^x$ meets the y-axis at the point P and the x-axis at the point Q.

a Find an equation for the normal to the curve at P.

b Find an equation for the tangent to the curve at Q.

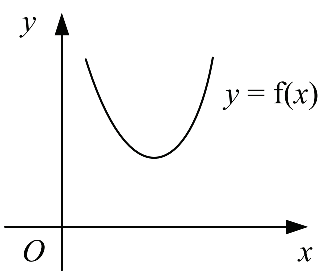
The normal to the curve at P meets the tangent to the curve at Q at the point R.

The x-coordinate of R is $a \ln 2 + b$, where a and b are rational constants.

c Show that $a = \frac{8}{5}$.

d Find the value of b.

10



The diagram shows a sketch of the curve $y = f(x)$ where

$$f : x \to 9x^4 - 16 \ln x, \quad x > 0.$$

Given that the set of values of x for which $f(x)$ is a decreasing function of x is $0 < x \leq k$, find the exact value of k.

1 Differentiate with respect to x

a $(x+3)^5$ **b** $(2x-1)^3$ **c** $(8-x)^7$ **d** $2(3x+4)^6$

e $(6-5x)^4$ **f** $\frac{1}{x-2}$ **g** $\frac{4}{(2x+3)^3}$ **h** $\frac{1}{(7-3x)^2}$

2 Differentiate with respect to t

a $2e^{3t}$ **b** $\sqrt{4t-1}$ **c** $5\ln 2t$ **d** $(8-3t)^{\frac{3}{2}}$

e $3\ln(6t+1)$ **f** $\frac{1}{2}e^{5t+4}$ **g** $\frac{6}{\sqrt[3]{2t-5}}$ **h** $2\ln(3-\frac{1}{4}t)$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y=(3x-1)^4$ **b** $y=4\ln(1+2x)$ **c** $y=\sqrt{5-2x}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x)=x^2-6\ln 2x$, $x=3$ **b** $f(x)=3+2x-e^{x-2}$, $x=2$

c $f(x)=(2-5x)^4$, $x=\frac{1}{2}$ **d** $f(x)=\frac{4}{x+5}$, $x=-1$

5 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x)=4\sqrt{3x+15}$, $f'(x)=2$ **b** $f(x)=x^2-\ln(x-2)$, $f'(x)=5$

6 Differentiate with respect to x

a $(x^2-4)^3$ **b** $2(3x^2+1)^6$ **c** $\ln(3+2x^2)$ **d** $(2+x)^3(2-x)^3$

e $\left(\frac{x^4+6}{2}\right)^8$ **f** $\frac{1}{\sqrt{3-x^2}}$ **g** $4+7e^{x^2}$ **h** $(1-5x+x^3)^4$

i $3\ln(4-\sqrt{x})$ **j** $(e^{4x}+2)^7$ **k** $\frac{1}{5+4\sqrt{x}}$ **l** $(\frac{2}{x}-x)^5$

7 Find the coordinates of any stationary points on each curve.

a $y=(2x-3)^5$ **b** $y=(x^2-4)^3$ **c** $y=8x-e^{2x}$

d $y=\sqrt{1+2x^2}$ **e** $y=2\ln(x-x^2)$ **f** $y=4x+\frac{1}{x-3}$

8 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinat

a $y=(3x-7)^4$, $x=2$ **b** $y=2+\ln(1+4x)$, $x=0$

c $y=\frac{9}{x^2+2}$, $x=1$ **d** $y=\sqrt{5x-1}$, $x=\frac{1}{4}$

9 Find an equation for the normal to each curve at the point on the curve with the given x-coordinat

a $y=e^{4-x^2}-10$, $x=-2$ **b** $y=(1-2x^2)^3$, $x=\frac{1}{2}$

c $y=\frac{1}{2-\ln x}$, $x=1$ **d** $y=6e^{\frac{x}{3}}$, $x=3$

1 Find an equation for the tangent to the curve with equation $y = x^2 + \ln(4x - 1)$ at the point on the curve where $x = \frac{1}{2}$.

2 A curve has the equation $y = \sqrt{8 - e^{2x}}$.

The point P on the curve has y-coordinate 2.

a Find the x-coordinate of P.

b Show that the tangent to the curve at P has equation

$$2x + y = 2 + \ln 4.$$

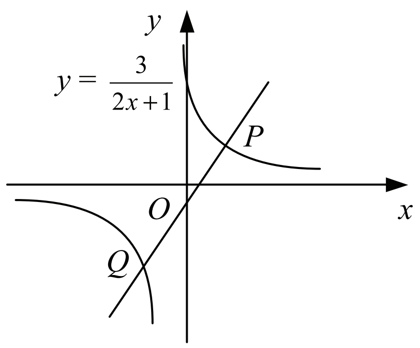
3 A curve has the equation $y = 2x + 1 + \ln(4 - 2x)$, $x < 2$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve.

c Determine the nature of this stationary point.

4



The diagram shows the curve with equation $y = \frac{3}{2x+1}$.

a Find an equation for the normal to the curve at the point P (1, 1).

The normal to the curve at P intersects the curve again at the point Q.

b Find the exact coordinates of Q.

5 A quantity N is increasing such that at time t seconds,

$$N = ae^{kt}.$$

Given that at time $t = 0$, $N = 20$ and that at time $t = 8$, $N = 60$, find

a the values of the constants a and k,

b the value of N when $t = 12$,

c the rate at which N is increasing when $t = 12$.

6 $$f(x) \equiv (5 - 2x^2)^3.$$

a Find $f'(x)$.

b Find the coordinates of the stationary points of the curve $y = f(x)$.

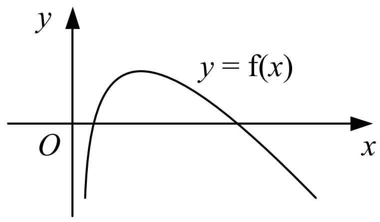
c Find the equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate $\frac{3}{2}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

7 A curve has the equation $y = 4x - \frac{1}{2}e^{2x}$.

a Find the coordinates of the stationary point of the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

8



The diagram shows the curve $y = f(x)$ where $f(x) = 3 \ln 5x - 2x$, $x > 0$.

a Find $f'(x)$.

b Find the x-coordinate of the point on the curve at which the gradient of the normal to the curve is $-\frac{1}{4}$.

c Find the coordinates of the maximum turning point of the curve.

d Write down the set of values of x for which $f(x)$ is a decreasing function.

9 The curve C has the equation $y = \sqrt{x^2 + 3}$.

a Find an equation for the tangent to C at the point A $(-1, 2)$.

b Find an equation for the normal to C at the point B $(1, 2)$.

c Find the x-coordinate of the point where the tangent to C at A meets the normal to C at B.

10 A bucket of hot water is placed outside and allowed to cool. The surface temperature of the water, T °C, after t minutes is given by

$$T = 20 + 60e^{-kt},$$

where k is a positive constant.

a State the initial surface temperature of the water.

b State, with a reason, the air temperature around the bucket.

Given that $T = 30$ when $t = 25$,

c find the value of k,

d find the rate at which the surface temperature of the water is decreasing when $t = 40$.

11
$$f(x) \equiv x^2 - 7x + 4 \ln \left(\frac{x}{2}\right), \quad x > 0.$$

a Solve the equation $f'(x) = 0$, giving your answers correct to 2 decimal places.

b Find an equation for the tangent to the curve $y = f(x)$ at the point on the curve where $x = 2$.

12 A curve has the equation $y = x^2 - \frac{8}{x-1}$.

a Show that the x-coordinate of any stationary point of the curve satisfies the equation

$$x^3 - 2x^2 + x + 4 = 0.$$

b Hence, show that the curve has exactly one stationary point and find its coordinates.

c Determine the nature of this stationary point.

1 Given that $f(x) = x(x + 2)^3$, find $f'(x)$

a by first expanding $f(x)$, **b** using the product rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a xe^x **b** $x(x + 1)^5$ **c** $x \ln x$ **d** $x^2(x - 1)^3$

e $x^3 \ln 2x$ **f** x^2e^{-x} **g** $2x^4(5 + x)^3$ **h** $x^2(x - 3)^4$

3 Find $\frac{dy}{dx}$, simplifying your answer in each case.

a $y = x(2x - 1)^3$ **b** $y = 3x^4e^{2x + 3}$ **c** $y = x\sqrt{x-1}$

d $y = x^2 \ln (x + 6)$ **e** $y = x(1 - 5x)^4$ **f** $y = (x + 2)(x - 3)^3$

g $y = x^{\frac{4}{3}}e^{3x}$ **h** $y = (x + 1) \ln (x^2 - 1)$ **i** $y = x^2\sqrt{3x+1}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 4xe^{3x}$, $x = 0$ **b** $f(x) = 2x(x^2 + 2)^3$, $x = -1$

c $f(x) = (5x - 4) \ln 3x$, $x = \frac{1}{3}$ **d** $f(x) = x^{\frac{1}{2}}(1 - 2x)^3$, $x = \frac{1}{4}$

5 Find the coordinates of any stationary points on each curve.

a $y = xe^{2x}$ **b** $y = x(x - 4)^3$ **c** $y = x^2(2x - 3)^4$

d $y = x\sqrt{x+12}$ **e** $y = 2 + x^2e^{-4x}$ **f** $y = (1 - 3x)(3 - x)^3$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinat

a $y = x(x - 2)^4$, $x = 1$ **b** $y = 3x^2e^x$, $x = 1$

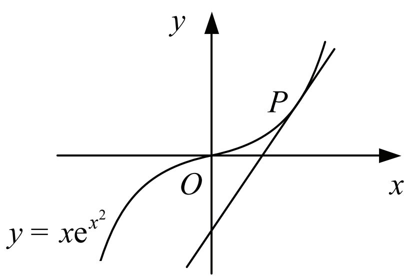
c $y = (4x - 1) \ln 2x$, $x = \frac{1}{2}$ **d** $y = x^2\sqrt{x+6}$, $x = -2$

7 Find an equation for the normal to each curve at the point on the curve with the given x-coordinat
Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2(2 - x)^3$, $x = 1$ **b** $y = x \ln (3x - 5)$, $x = 2$

c $y = (x^2 - 1)e^{3x}$, $x = 0$ **d** $y = x\sqrt{x-4}$, $x = 8$

8



The diagram shows part of the curve with equation $y = xe^{x^2}$ and the tangent to the curve at the point P with x-coordinate 1.

a Find an equation for the tangent to the curve at P.

b Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3}e$.

C3 DIFFERENTIATION Worksheet F

1 Given that $f(x) = \dfrac{x}{x+2}$, find $f'(x)$

a using the product rule, **b** using the quotient rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a $\dfrac{4x}{1-3x}$ **b** $\dfrac{e^x}{x-4}$ **c** $\dfrac{x+1}{2x+3}$ **d** $\dfrac{\ln x}{2x}$

e $\dfrac{x}{2-x^2}$ **f** $\dfrac{\sqrt{x}}{3x+2}$ **g** $\dfrac{e^{2x}}{1-e^{2x}}$ **h** $\dfrac{2x+1}{\sqrt{x-3}}$

3 Find $\dfrac{dy}{dx}$, simplifying your answer in each case.

a $y = \dfrac{x^2}{x+4}$ **b** $y = \dfrac{\sqrt{x-4}}{2x^2}$ **c** $y = \dfrac{2e^x+1}{1-3e^x}$

d $y = \dfrac{1-x}{x^3+2}$ **e** $y = \dfrac{\ln(3x-1)}{x+2}$ **f** $y = \sqrt{\dfrac{x+1}{x+3}}$

4 Find the coordinates of any stationary points on each curve.

a $y = \dfrac{x^2}{3-x}$ **b** $y = \dfrac{e^{4x}}{2x-1}$ **c** $y = \dfrac{x+5}{\sqrt{2x+1}}$

d $y = \dfrac{\ln 3x}{2x}$ **e** $y = \left(\dfrac{x+1}{x-2}\right)^2$ **f** $y = \dfrac{x^2-3}{x+2}$

5 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinat

a $y = \dfrac{2x}{3-x}$, $x = 2$ **b** $y = \dfrac{e^x+3}{e^x+1}$, $x = 0$

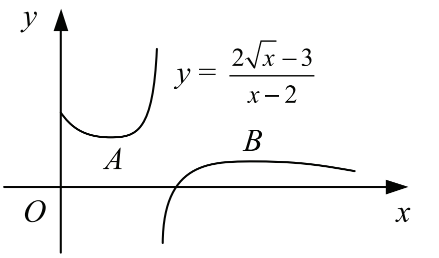
c $y = \dfrac{\sqrt{x}}{5-x}$, $x = 4$ **d** $y = \dfrac{3x+4}{x^2+1}$, $x = -1$

6 Find an equation for the normal to each curve at the point on the curve with the given x-coordinat
Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = \dfrac{1-x}{3x+1}$, $x = 1$ **b** $y = \dfrac{4x}{\sqrt{2-x}}$, $x = -2$

c $y = \dfrac{\ln(2x-5)}{3x-5}$, $x = 3$ **d** $y = \dfrac{x}{x^3-4}$, $x = 2$

7



The diagram shows part of the curve $y = \dfrac{2\sqrt{x}-3}{x-2}$ which is stationary at the points A and B.

a Show that the x-coordinates of A and B satisfy the equation $x - 3\sqrt{x} + 2 = 0$.

b Hence, find the coordinates of A and B.

1 A curve has the equation $y = x^2(2-x)^3$ and passes through the point A (1, 1).

a Find an equation for the tangent to the curve at A.

b Show that the normal to the curve at A passes through the origin.

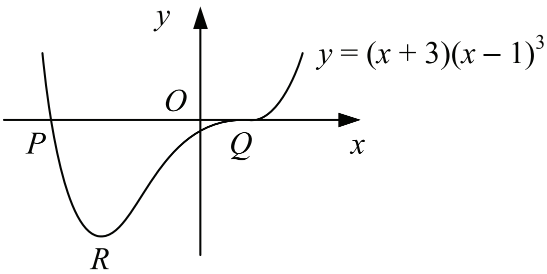
2 A curve has the equation $y = \frac{x}{2x+3}$.

a Find an equation for the tangent to the curve at the point P (−1, −1).

b Find an equation for the normal to the curve at the origin, O.

c Find the coordinates of the point where the tangent to the curve at P meets the normal to the curve at O.

3



The diagram shows the curve with equation $y = (x+3)(x-1)^3$ which crosses the x-axis at the points P and Q and has a minimum at the point R.

a Write down the coordinates of P and Q.

b Find the coordinates of R.

4 Given that $y = x\sqrt{4x+1}$,

a show that $\frac{dy}{dx} = \frac{6x+1}{\sqrt{4x+1}}$,

b solve the equation $\frac{dy}{dx} - 5y = 0$.

5 A curve has the equation $y = \frac{2(x-1)}{x^2+3}$ and crosses the x-axis at the point A.

a Show that the normal to the curve at A has the equation $y = 2 - 2x$.

b Find the coordinates of any stationary points on the curve.

6 $$f(x) \equiv x^{\frac{3}{2}}(x-3)^3, \; x > 0.$$

a Show that

$$f'(x) = kx^{\frac{1}{2}}(x-1)(x-3)^2,$$

where k is a constant to be found.

b Hence, find the coordinates of the stationary points of the curve $y = f(x)$.

7 $$f(x) = x\sqrt{2x+12}, \; x \geq -6.$$

a Find $f'(x)$ and show that $f''(x) = \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$.

b Find the coordinates of the turning point of the curve $y = f(x)$ and determine its nature.

1 Differentiate with respect to x

a $\cos x$ **b** $5 \sin x$ **c** $\cos 3x$ **d** $\sin \frac{1}{4}x$

e $\sin (x + 1)$ **f** $\cos (3x - 2)$ **g** $4 \sin (\frac{\pi}{3} - x)$ **h** $\cos (\frac{1}{2}x + \frac{\pi}{6})$

i $\sin^2 x$ **j** $2 \cos^3 x$ **k** $\cos^2 (x - 1)$ **l** $\sin^4 2x$

2 Use the derivatives of $\sin x$ and $\cos x$ to show that

a $\frac{d}{dx}(\tan x) = \sec^2 x$ **b** $\frac{d}{dx}(\sec x) = \sec x \tan x$

c $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$ **d** $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

3 Differentiate with respect to t

a $\cot 2t$ **b** $\sec (t + 2)$ **c** $\tan (4t - 3)$ **d** $\text{cosec } 3t$

e $\tan^2 t$ **f** $3 \text{ cosec } (t + \frac{\pi}{6})$ **g** $\cot^3 t$ **h** $4 \sec \frac{1}{2}t$

i $\cot (2t - 3)$ **j** $\sec^2 2t$ **k** $\frac{1}{2} \tan (\pi - 4t)$ **l** $\text{cosec}^2 (3t + 1)$

4 Differentiate with respect to x

a $\ln (\sin x)$ **b** $6e^{\tan x}$ **c** $\sqrt{\cos 2x}$ **d** $e^{\sin 3x}$

e $2 \cot x^2$ **f** $\sqrt{\sec x}$ **g** $3e^{-\text{cosec } 2x}$ **h** $\ln (\tan 4x)$

5 Find the coordinates of any stationary points on each curve in the interval $0 \le x \le 2\pi$.

a $y = x + 2 \sin x$ **b** $y = 2 \sec x - \tan x$ **c** $y = \sin x + \cos 2x$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinat

a $y = 1 + \sin 2x, \quad x = 0$ **b** $y = \cos x, \quad x = \frac{\pi}{3}$

c $y = \tan 3x, \quad x = \frac{\pi}{4}$ **d** $y = \text{cosec } x - 2 \sin x, \quad x = \frac{\pi}{6}$

7 Differentiate with respect to x

a $x \sin x$ **b** $\frac{\cos 2x}{x}$ **c** $e^x \cos x$ **d** $\sin x \cos x$

e $x^2 \text{ cosec } x$ **f** $\sec x \tan x$ **g** $\frac{x}{\tan x}$ **h** $\frac{\sin 2x}{e^{3x}}$

i $\cos^2 x \cot x$ **j** $\frac{\sec 2x}{x^2}$ **k** $x \tan^2 4x$ **l** $\frac{\sin x}{\cos 2x}$

8 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = \sin 3x \cos 5x, \quad x = \frac{\pi}{4}$ **b** $f(x) = \tan 2x \sin x, \quad x = \frac{\pi}{3}$

c $f(x) = \frac{\ln(2\cos x)}{\sin x}, \quad x = \frac{\pi}{3}$ **d** $f(x) = \sin^2 x \cos^3 x, \quad x = \frac{\pi}{6}$

9 Find an equation for the normal to the curve $y = 3 + x\cos 2x$ at the point where it crosses the y-axis.

10 A curve has the equation $y = \frac{2+\sin x}{1-\sin x}$, $0 \le x \le 2\pi$, $x \ne \frac{\pi}{2}$.

a Find and simplify an expression for $\frac{dy}{dx}$.

b Find the coordinates of the turning point of the curve.

c Show that the tangent to the curve at the point P, with x-coordinate $\frac{\pi}{6}$, has equation

$$y = 6\sqrt{3}x + 5 - \sqrt{3}\pi.$$

11 A curve has the equation $y = e^{-x}\sin x$.

a Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the exact coordinates of the stationary points of the curve in the interval $-\pi \le x \le \pi$ and determine their nature.

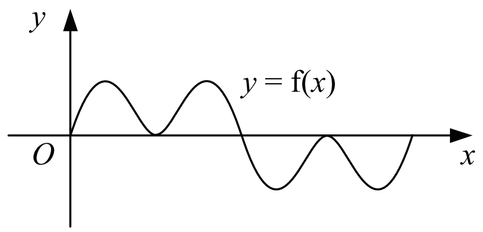
12 The curve C has the equation $y = x\sec x$.

a Show that the x-coordinate of any stationary point of C must satisfy the equation

$$1 + x\tan x = 0.$$

b By sketching two suitable graphs on the same set of axes, deduce the number of stationary points C has in the interval $0 \le x \le 2\pi$.

13



The diagram shows the curve $y = f(x)$ in the interval $0 \le x \le 2\pi$, where

$$f(x) \equiv \cos x \sin 2x.$$

a Show that $f'(x) = 2\cos x\,(1 - 3\sin^2 x)$.

b Find the x-coordinates of the stationary points of the curve in the interval $0 \le x \le 2\pi$.

c Show that the maximum value of f(x) in the interval $0 \le x \le 2\pi$ is $\frac{4}{9}\sqrt{3}$.

d Explain why this is the maximum value of f(x) for all real values of x.

14 A curve has the equation $y = \text{cosec}\,(x - \frac{\pi}{6})$ and crosses the y-axis at the point P.

a Find an equation for the normal to the curve at P.

The point Q on the curve has x-coordinate $\frac{\pi}{3}$.

b Find an equation for the tangent to the curve at Q.

The normal to the curve at P and the tangent to the curve at Q intersect at the point R.

c Show that the x-coordinate of R is given by $\frac{8\sqrt{3}+4\pi}{13}$.