Solomon Practice Paper

Pure Mathematics 4G

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	7	
2	7	
3	9	
4	11	
5	12	
6	12	
7	17	
Total:	75	

How I can achieve better:

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BY NC ND

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1. Find the set of values of x for which [7]

$$\frac{x^2 - 12}{x} \geq 1.$$

2. Show that the sum of the first n terms of the series [7]

$$5^2 + 9^2 + 13^3 + 17^2 + \dots$$

is given by $\frac{1}{3}n(16n^2 + 36n + 23)$.

3.

$$\mathrm{f}(x) \equiv x^3 - 5x^2 + 2.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval $[0, 1]$. [2]

(b) Use the Newton-Raphson method with initial value $x = 0.5$ to find a value for α which is correct to 2 decimal places. [5]

(c) Give a reason why the Newton-Raphson method fails if an initial value of $x = 0$ is used in part (b). [2]

Total: 9

4. The complex number z is given by

$$z = \frac{1 + \mathbf{i}\sqrt{3}}{1 - \mathbf{i}\sqrt{3}}.$$

(a) Show that z can be expressed in the form $\lambda(1 - \mathbf{i}\sqrt{3})$ where λ is a rational number which you should find. [4]

(b) Find the modulus and argument of z. [3]

(c) Hence, or otherwise, find the modulus and argument of [4]

$$\left(\frac{1 + \mathbf{i}\sqrt{3}}{1 - \mathbf{i}\sqrt{3}}\right)^4.$$

Total: 11

5. (a) Find the values of p and q such that $y = p\sin(x) + q\cos(x)$ is a particular integral of the differential equation [7]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = \sin(x).$$

(b) Find the general solution of this differential equation. [5]

Total: 12

6. (a) Show that [3]

$$\int 2\cot(x)\,\mathrm{d}x = \ln\left(\sin^2(x)\right) + c,$$

where c is an arbitrary constant.

(b) Find the general solution of the differential equation [5]

$$\sin(x)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\cos(x) = 1.$$

Given that $y = 0$ when $x = \frac{\pi}{4}$,

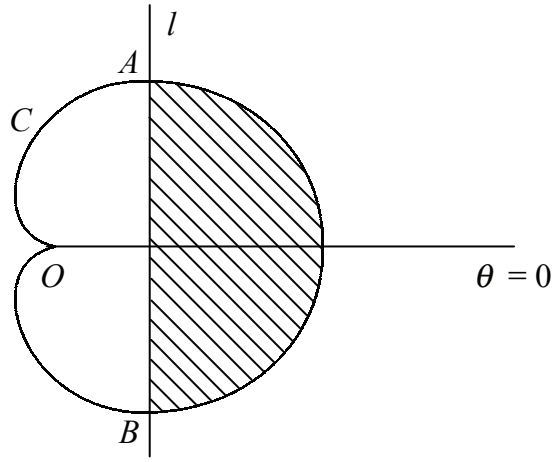
(c) show that when $x = \frac{\pi}{3}$, [4]

$$y = \frac{2}{3}\left(\sqrt{2} - 1\right).$$

Total: 12

7. Figure shows the curve C with polar equation

$$r = 2(1 + \cos(\theta)), \quad -\pi < \theta \leq \pi,$$



and the line l with polar equation

$$r\cos(\theta) = \frac{3}{2},$$

referred to the pole O and initial line $\theta = 0$.

(a) Find the polar coordinates of the points A and B, where l intersects C. [6]

(b) Show that the area of triangle OAB is $\frac{9\sqrt{3}}{4}$. [3]

(c) Hence find the area of the shaded region bounded by C and l. [8]

Total: 17