**Solomon Practice Paper**

**Pure Mathematics 4F**

**Time allowed:** 90 **minutes**

**Centre:** **www.CasperYC.club**

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 4 | |
| 2 | 7 | |
| 3 | 7 | |
| 4 | 7 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 14 | |
| 8 | 16 | |
| Total: | 75 | |

**How I can achieve better:**

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BY NC ND

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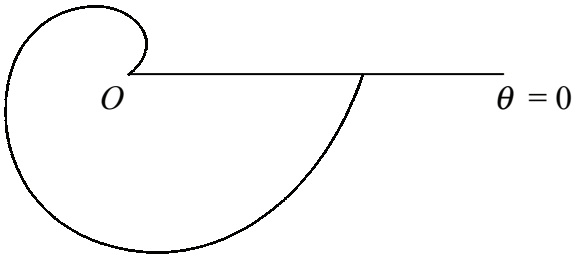
1. Figure shows the curve with polar equation [4]

$$r = a\theta, \quad 0 \leq \theta < 2\pi, \quad a > 0.$$




Find the area of the finite region bounded by the curve and the initial line $\theta = 0$.

2. Find the set of values of $x$ for which [7]

$$\frac{(x-1)(x+2)}{x+4} > 4.$$

3.

$$\mathrm{f}(x) = 3x^5 - 7x^2 + 3.$$

(a) Show that there is a root, $\alpha$, of the equation $\mathrm{f}(x) = 0$ in the interval $[0, 1]$. [2]

(b) Use linear interpolation once on the interval $[0, 1]$ to estimate the value of $\alpha$. [2]

There is another root, $\beta$, of the equation $\mathrm{f}(x) = 0$ close to $-0.62$.

(c) Use the Newton-Raphson method once to obtain a second approximation to $\beta$ , giving your answer correct to 3 decimal places. [3]

Total: 7

4. The Cartesian equation of the curve $C$ is

$$(x^2 + y^2)^2 = a^2(x^2 - y^2).$$

(a) Show that, in polar coordinates, the equation of curve $C$ can be written as [4]

$$r^2 = a^2 \cos(2\theta).$$

(b) Sketch the curve $C$ for $0 \leq \theta < 2\pi$. [3]

Total: 7

5. (a) Show that the substitution $y = \dfrac{1}{u}$ transforms the differential equation [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} - xy^2 = 0 \qquad (\star)$$

into the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{u}{x} + x = 0.$$

(b) Hence find the solution of differential equation $\star$ such that $y = 1$ when $x = 1$, giving your answer in the form $y = \mathrm{f}(x)$. [7]

Total: 10

6. (a) Find $\sum_{r=n+1}^{2n} r^2$ in terms of $n$. [4]

   (b) Hence, or otherwise, show that [6]

$$4 \leq \frac{\sum_{r=n+1}^{2n} r^2}{\sum_{r=1}^{n} r^2} < 7$$

   for all positive integer values of $n$.

Total: 10

7. A particle moves along the $x$-axis such that at time $t$ its $x$-coordinate satisfies the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 5\frac{\mathrm{d}x}{\mathrm{d}t} - 3x = 20\sin(t).$$

(a) Find the general solution of this differential equation. [10]

Initially the particle is at $x = 5$.

Given that the particle's $x$-coordinate remains finite as $t \to \infty$,

(b) find an expression for $x$ in terms of $t$. [4]

Total: 14

8. The complex numbers $z_1$ and $z_2$ are given by

$$z_1 = \frac{1+\mathbf{i}}{1-\mathbf{i}}, \quad \text{and} \quad z_2 = \frac{\sqrt{2}}{1-\mathbf{i}}.$$

(a) Find $z_1$ in the form $a + \mathbf{i}b$ where $a$ and $b$ are real. [2]

(b) Write down the modulus and argument of $z_1$. [2]

(c) Find the modulus and argument of $z_2$. [4]

(d) Show the points representing $z_1$, $z_2$ and $z_1 + z_2$ on the same Argand diagram, and hence find the exact value of $\tan\left(\frac{3\pi}{8}\right)$. [8]

Total: 16