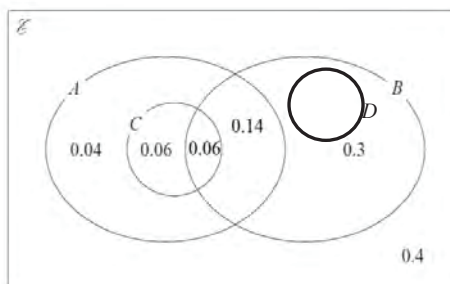
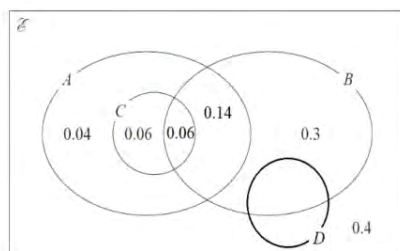
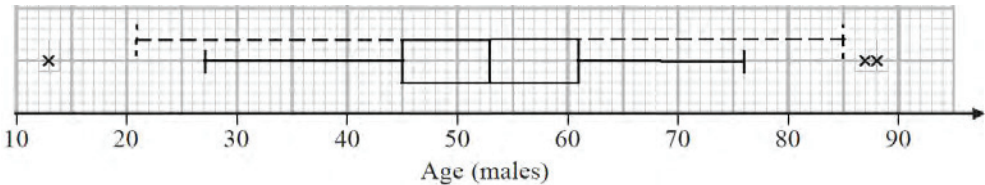
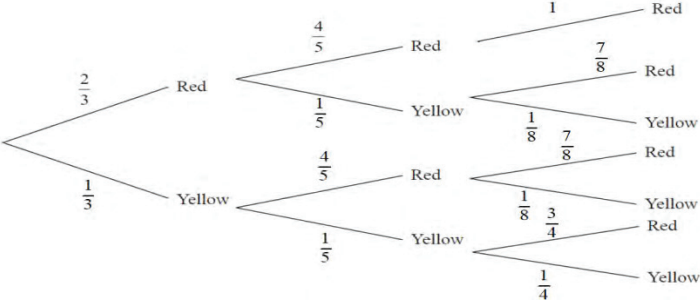


Question Number	Scheme	Marks
1 (a)	[Sum of probs = 1 gives $p + q = 0.2$ and] so $P(B) = \underline{0.5}$	B1 (1)
(b)	e.g. $P(A) = 0.3$ or $0.1 +$ "their value for $p + q$ ", $P(A \cap B) = 0.2$ or "their value for $p + q$ ", and $[P(A) \times P(B) =] 0.3 \times "0.5" [= "0.15"]$ $0.15 \neq 0.2$ so $[A$ and $B$ are] not independent	M1 A1 (2)
(c)	$[P(C   B) =] \frac{p}{"0.5"} = p + 0.06$ (o.e.) $[2p = p + 0.06$ so ] $\underline{p = 0.06}$ [Use of $p + q = 0.2$ gives] $\underline{q = 0.14}$	M1 A1 A1 (3)
(d)	Suitable event $D$ drawn. [See Venn diagrams below]	B1 (1)
<b>Notes</b>		
(a)	B1 for 0.5 or exact equivalent	
(b)	M1 for sight of correct probabilities for $P(A)$ and $P(A \cap B)$ <b>clearly labelled</b> , $0.3 \times "0.5"$ seen or $P(A) \times P(B) = 0.15$ Allow $0.04 + 0.06 + 0.2$ for $P(A)$ if clearly labelled $P(A \cap B)$ may be stated in part (a) $P(B)$ can fit from (a) eg $P(A) = "0.5" - 0.3 - 0.2$ May see $P(B   A) = \frac{2}{3}$ and compared with $P(B)$ or $P(A   B) = 0.4$ and $P(A) = 0.3$	
ALT	A1 For all the correct probabilities and calculations, a comparison and correct conclusion. We need to see 0.15 but will accept $P(A \cap B) \neq P(A) \times P(B)$ instead of $0.15 \neq 0.2$ for comparison SC Allow M1A0 for $P(A) = 0.1 + p + q$ ; $P(A \cap B) = p + q$ clearly labelled and $0.5 \times (0.1 + p + q)$ or $(p + q + 0.3)(0.1 + p + q)$ given.	
(c)	M1 fit their $P(B)$ from part (a). For a correct equation in $p$ or $q$ based on the given statement. NB equation in terms of $q$ is $\frac{0.2 - q}{"0.5"} = 0.26 - q$ (o.e.) Allow $\frac{p}{0.3 + p + q} = p + 0.06$	
Ans only	1 <sup>st</sup> A1 for ( $p =$ ) 0.06 2 <sup>nd</sup> A1 for ( $q =$ ) 0.14 ( $p =$ ) 0.06 and ( $q =$ ) 0.14 3/3	
(d)	B1 for a suitable event $D$ drawn that has an intersection with $B$ but not with $A$ . Condone if not labelled $D$	



Question Number	Scheme	Marks
<p>2 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	$[S_{xp} = ] 2347 - \frac{93 \times 273}{12} \text{ or } 2347 - \frac{25\,389}{12} [= 231.25] (*)$ $[S_{pp} = ] 6602.72 - \frac{273^2}{12} = [391.97]$ $[r = ] \frac{231.25}{\sqrt{148.25 \times 391.97}}$ $= 0.959307... \text{ awrt } \underline{0.959}$ $\left[ b = \frac{S_{xp}}{S_{xx}} = \right] \frac{231.25}{148.25} [= 1.559865...]$ $a = \frac{273}{12} - "1.56" \times \frac{93}{12} \text{ or } 22.75 - "1.56" \times 7.75 [= 10.66...]$ <p><math>b = \text{awrt } 1.6 \text{ or } a = \text{awrt } 11</math></p> $\underline{p = 10.7 + 1.56x}$ <p>e.g. each extra <b>employee</b> costs the company (on average)["\$"]156" a year in <b>paper</b></p> $[\text{New } p = ] 0.8 \times "10.66..." + \frac{"1.559..." \times 93}{2} \times \frac{93}{12} [= 14.573...]$ <p>[compared with <math>\bar{p} = 22.75</math>] so percentage saving is <math>\frac{22.75 - 14.573...}{22.75} [\times 100]</math></p> $= 35.94... \text{ awrt } \underline{36[\%]}$	<p>B1cso</p> <p>(1)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p> <p>B1</p> <p>(1)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p>
<b>Notes</b>		<b>[12]</b>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>B1 for either correct expression [don't need = 231.25]</p> <p>1<sup>st</sup> M1 for attempt at correct expression for <math>S_{pp}</math> Allow one transcription error e.g. 6620.72 May be seen in part (a)</p> <p>If no correct expression seen allow <math>S_{pp} = \text{awrt } 392</math> or correctly placed in formula for <math>r</math></p> <p>2<sup>nd</sup> M1 for a correct expression for <math>r</math>, ft their <math>S_{pp}</math></p> <p>A1 for awrt 0.959</p> <p>1<sup>st</sup> M1 for a correct expression for <math>b</math> (may be implied by a correct value of awrt 1.56)</p> <p>2<sup>nd</sup> M1 for a correct expression for <math>a</math> ft their value for <math>b</math>. May be implied by awrt 10.7</p> <p>1<sup>st</sup> A1 for <math>b = \text{awrt } 1.6 \text{ or } a = \text{awrt } 11</math></p> <p>2<sup>nd</sup> A1 for correct equation in <math>p</math> and <math>x</math> with <math>b = \text{awrt } 1.56</math> and <math>a = \text{awrt } 10.7</math></p> <p>B1 for a suitable contextual comment that mentions their value of <math>b</math></p> <p>Allow multiples eg every extra 100 employees costs the company "\$15600". Condone missing \$ sign or use of £. Do not allow " \$1.56" for 1 person unless indicates in 100's</p> <p>1<sup>st</sup> M1 for a correct expression for average value of <math>p</math> using new model [ft their <math>a</math> and <math>b</math>]</p> <p>2<sup>nd</sup> M1 for correct percentage saving calculation using 22.75 (e.g. <math>\frac{14.573...}{22.75} [\times 100]</math>) Allow use of</p> $"10.7" + 1.56 \times \frac{93}{12} [\approx 22.79] \text{ for } 22.75 \text{ May be implied by correct answer.}$ <p>A1 for awrt 36</p> <p>SC use of 93 throughout part (e) rather than 7.75 leading to awrt 48 or 0.48 (they will need to use the regression line from part(c) to calculate the original value) gains M0M1A0</p> <p>SC use of 93 in part(c) Answer of 36% gains M1M1A1, 64% or 0.64 gains M1M1A0.</p>	

Question Number	Scheme	Marks
3. (a)	[Median =] <b>53</b>	B1 (1)
(b)	$Q_1 = 45$ $Q_3 = 61$ [IQR =] $61 - 45 = \mathbf{16}$ (*)	M1 A1cso (2)
(c)	$Q_1 - 1.5 \times (\text{IQR}) = 45 - 1.5 \times 16 [= \mathbf{21}]$ or $Q_3 + 1.5 \times (\text{IQR}) = 61 + 1.5 \times 16 [= \mathbf{85}]$ Outliers are $< \mathbf{21}$ or $> \mathbf{85}$ So there are three outliers at <b>13, 87 and 88</b>	M1 A1ft A1 (3)
(d)		M1 A1ft A1 A1 (4)
(e)	e.g. the females are generally older than the men as median is higher ( $67 > 53$ )	B1 (1)
(f)(i)	No change to box plot means one in each section so granddaughter <b>[34~56]</b>	B1
(ii)	Eldest daughter in range [67~72] or Anja's age [72~93] Since Anja 23 years older than eldest daughter Anja in range <b>[90~93]</b>	M1 A1 (3)
<b>Notes</b>		<b>[14]</b>
(a)	B1 for 53	
(b)	M1 for an attempt at both <b>and</b> at least one correct. No need to be labelled.	
	A1cso for both correct quartiles seen <b>and</b> $61 - 45$ leading to 16	
(c)	M1 for attempting at least one of the limits. Can fit their quartiles. May be implied by 85 or 21 1 <sup>st</sup> A1ft for both outlier limits correct or correct ft using their quartiles 2 <sup>nd</sup> A1 for identifying the three outliers at 13, 87, 88 (dep on seeing both correct limits)	
(d)	M1 for drawing a box with only two whiskers one at each end 1 <sup>st</sup> A1ft for $Q_1, Q_2$ and $Q_3$ as a correctly drawn box (ft their values for $Q_1, Q_2$ and $Q_3$ ) 2 <sup>nd</sup> A1 for upper whisker ending at 76 (or 85) <b>and</b> lower whisker ending at 27 (or 21) <b>NB Must be correctly paired.</b> 3 <sup>rd</sup> A1 for the 3 outliers correctly shown (accuracy - half a small square throughout) SC fully correct but with both whiskers correct on each side. M0A1A0A1	
(e)	B1 for a correct comment, <b>referring to ages</b> , with reference to a <b>correctly named statistic</b> . Must <b>include the figures</b> compared. eg Females older than men and comparison of median, upper quartile or lower quartile, allow $Q_1, Q_2$ and $Q_3$ with their figures which must agree with the statement. eg Males ages more spread out than female and comparison of ranges with males = 75 and females = 73 eg Females older than males since Males are symmetrical [ $Q_3 - Q_2 : Q_2 - Q_1$ ] 8 : 8 Female are negative <b>skew</b> 5 : 11 <b>NB</b> use of mean/ IQR/ minimum/ maximum is B0. Ignore incorrect comments.	
(f)(i)	B1 for deducing granddaughter is at or below lower quartile but not below 34 Allow any reasonable adjustment for her mother's age, {34 to x} where $35 \leq x \leq 56$	
(ii)	M1 Suitable range for eldest daughter or Anja above upper quartile. Ignore any incorrect upper limit. May be implied by a correct range. A1 for a range of [90~93] for Anja's age	

Question Number	Scheme	Marks										
4. (a)	 <p>(b) [Cases RYY or YRY or YR] <math>\text{Prob} = \frac{2}{3} \times \frac{1}{5} \times \frac{1}{8} + \frac{1}{3} \times \frac{4}{5} \times \frac{1}{8} + \frac{1}{3} \times \frac{1}{5} \times \frac{6}{8}</math>  <math>[= \frac{1}{120}(2+4+6) \text{ or } (0.0166... + 0.033... + 0.05)] = \frac{12}{120} \text{ or } \frac{1}{10} (*)</math></p> <p>(c) <math>[P(RYY   RYY \text{ or } YRY \text{ or } YR) = ] \frac{\frac{2}{3} \times \frac{1}{5} \times \frac{1}{8}}{\frac{1}{10}} = \frac{1}{6}</math></p> <p>(d)</p> <table border="1" data-bbox="279 1008 1284 1108"> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>P(X=x)</math></td> <td><math>\frac{64}{120}</math> or <math>\frac{8}{15}</math></td> <td><math>\frac{42}{120}</math> or <math>\frac{7}{20}</math></td> <td><math>[\frac{1}{10}]</math></td> <td><math>\frac{2}{120}</math> or <math>\frac{1}{60}</math></td> </tr> </table> <p>(e) <math>[E(X) = ] \frac{1}{120}(0 + "42" + "2 \times 12" + "3 \times 2") = \frac{72}{120} \text{ or } \underline{0.6}</math></p>	$x$	0	1	2	3	$P(X=x)$	$\frac{64}{120}$ or $\frac{8}{15}$	$\frac{42}{120}$ or $\frac{7}{20}$	$[\frac{1}{10}]$	$\frac{2}{120}$ or $\frac{1}{60}$	<p>B1 B1 B1 (3)</p> <p>M1 A1ft A1*cso (3)</p> <p>M1 A1 (2)</p> <p>B1 M1 A1 (3)</p> <p>M1 A1 (2)</p> <p>[13]</p>
$x$	0	1	2	3								
$P(X=x)$	$\frac{64}{120}$ or $\frac{8}{15}$	$\frac{42}{120}$ or $\frac{7}{20}$	$[\frac{1}{10}]$	$\frac{2}{120}$ or $\frac{1}{60}$								
<b>Notes</b>												
(a)	<p>1<sup>st</sup> B1 completing the structure of branches: 2, 4 then 7 or 8 and suitable labels e.g. Y or R'  2<sup>nd</sup> B1 correct probabilities for at least bag A and bag B. Allow exact decimals  3<sup>rd</sup> B1 for a fully correct tree diagram. Condone missing 0 as probability. Allow exact decimals</p> <p>(b) 1<sup>st</sup> M1 for at least one correct product of 3 probabilities (ft their tree diagram)  1<sup>st</sup> A1ft for all 3 products of 3 probabilities added ( no extras) (ft their tree diag.)  2<sup>nd</sup> A1*cso for fully correct solution with no incorrect statements seen</p> <p>(c) M1 for a ratio of probabilities with denominator of 0.1 and numerator <math>\frac{1}{60}</math> oe or the product of 3 probabilities <b>seen</b> from their tree diagram representing P(RYY) provided num &lt; 0.1  A1 for <math>\frac{1}{6}</math> or exact equivalent</p> <p>(d) B1 for a correct sample space i.e. { 0, 1, 2, 3} Allow extras if they have a probability of 0.  M1 for at least 1 correct value of x and associated probability (excluding x = 2) [ft their tree]  A1 for a fully correct probability distribution</p> <p>(e) M1 for attempt at a correct expression (at least 2 correct ft part(d) non-zero products)  A1 for 0.6 or any exact equivalent</p>											

Question Number	Scheme	Marks								
<p>5. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>[By symmetry <math>E(Y)] = \underline{0}</math></p> <p><math>q + r + u = \frac{19}{30}</math></p> <p><math>2(q + r) + u = 1</math> [and attempt to solve e.g. <math>q + r = \dots</math>]</p> <p><math>u = \frac{8}{30} = \frac{4}{15}</math> (*)</p> <p><math>E(Y^2) = (-9)^2 \times q + (-5)^2 \times r + 5^2 \times r + 9^2 q</math> or <math>162q + 50r</math></p> <p><math>\text{Var}(Y) = 37 = E(Y^2) - "0"{}^2 \Rightarrow 37 = 162q + 50r</math> oe</p> <p>Solving with <math>q + r = \frac{11}{30}</math> oe e.g. <math>(162 - 50)q = 37 - \frac{55}{3}</math> or</p> <p><math>q = \frac{1}{6}</math> and <math>r = \frac{1}{5}</math></p> <p><math>Y = 0 \Rightarrow D = 12, D = \sqrt{12^2 + Y^2}; Y = \pm 5 \Rightarrow D = 13</math> or <math>Y = \pm 9 \Rightarrow D = 15</math></p> <table border="1" data-bbox="279 965 1046 1104"> <tr> <td><math>d</math></td> <td>12</td> <td>13</td> <td>15</td> </tr> <tr> <td><math>P(D = d)</math></td> <td><math>\frac{4}{15}</math></td> <td><math>\frac{6}{15}</math> or <math>\frac{2}{5}</math></td> <td><math>\frac{5}{15}</math> or <math>\frac{1}{3}</math></td> </tr> </table>	$d$	12	13	15	$P(D = d)$	$\frac{4}{15}$	$\frac{6}{15}$ or $\frac{2}{5}$	$\frac{5}{15}$ or $\frac{1}{3}$	<p>B1</p> <p>(1)</p> <p>M1</p> <p>M1</p> <p>A1*cso</p> <p>(3)</p> <p>M1</p> <p>dM1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>B1, M1;A1</p> <p>M1A1ftA1ft</p> <p>(6)</p> <p>[14]</p>
$d$	12	13	15							
$P(D = d)$	$\frac{4}{15}$	$\frac{6}{15}$ or $\frac{2}{5}$	$\frac{5}{15}$ or $\frac{1}{3}$							
<b>Notes</b>										
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>B1 for 0</p> <p>1<sup>st</sup> M1 for a correct equation in <math>q, r</math> and <math>u</math> using <math>F(0)</math></p> <p>2<sup>nd</sup> M1 for a second equation clearly based on sum of probs = 1 and an attempt to solve these 2 equations</p> <p>A1* cso correct value for <math>u</math> found with no incorrect working</p> <p>1<sup>st</sup> M1 for an attempt at <math>E(Y^2)</math> with at least 3 correct products seen. The negative numbers do not need to be in brackets</p> <p>2<sup>nd</sup> dM1 for attempt at correct equation in <math>q</math> and <math>r</math> using <math>\text{Var}(Y)</math> [ft their <math>E(Y)</math> and <math>E(Y^2)</math>] Condone missing subtraction of <math>0^2</math> if 0 in part(a)</p> <p>3<sup>rd</sup> M1 using <math>q + r = 11/30</math> (awrt 0.37) to attempt to solve two linear equations in <math>q</math> and <math>r</math> leading to equation in one variable. May be implied by correct answers.</p> <p>A1 for <math>q = \frac{1}{6}</math> and <math>r = \frac{1}{5}</math> or exact equivalents</p> <p>B1 for <math>D = 12</math></p> <p>1<sup>st</sup> M1 for use of Pythagoras to work out <math>D = 13</math> or 15</p> <p>1<sup>st</sup> A1 for <math>D = 13</math> and 15</p> <p>2<sup>nd</sup> M1 for a correct value of <math>D</math> and an associated probability. Allow two occurrences (for 15 and 13) which add to the appropriate probability.</p> <p>2<sup>nd</sup>A1ft for two correct values of <math>D</math> and associated probs ft their +ve <math>q</math> and <math>r</math> if <math>q + r = \frac{11}{30}</math> Allow two occurrences (for 15 and 13) which add to the appropriate probability.</p> <p>3<sup>rd</sup> A1ft for a fully correct probability distribution ft their +ve <math>q</math> and <math>r</math> if <math>q + r = \frac{11}{30}</math></p>									

Question Number	Scheme	Marks
6. (a)	$H \sim N(25.1, 5.5^2)$ $P(H > 30.8) = P\left(Z > \frac{30.8 - 25.1}{5.5}\right)$ or $P(Z > 1.03636\dots)$ $= 1 - 0.8508$ $= 0.1492$ or better (calc: 0.1500...)	M1 M1 A1cso (3)
(b)	$[P(H < y) = 0.05 \text{ implies}] \quad \frac{y - 25.1}{5.5} = -1.6449$ $y = 16.053\dots$ so range is awrt <b>16.1 ~ 30.8</b>	M1B1 A1 (3)
(c)(i)	$P(H < d) = 0.05 + 0.2 + 0.3 [= 0.55]$ $\frac{d - 25.1}{5.5} = 0.13$ (Calc 0.12566) $d = 0.13 \times 5.5 + 25.1 = 25.815$ (25.791... calc)	M1 M1 A1cso (3)
(ii)	$P(H < m) = 0.05 + 0.2 [= 0.25]$ $\frac{m - 25.1}{5.5} = -0.67$ (Calc 0.674489) $m = \text{awrt } \underline{21.4}$	M1 M1M1 A1 (4)
(d)	$\text{Height} = 2 \times "m" + 3 \times 25.8 + 3 \times 30.8 [+8]$ $= 220.6$ awrt <b>221</b> (cm)	M1 A1 (2)
	<b>Notes</b>	<b>[15]</b>
(a)	1 <sup>st</sup> M1 for standardising 30.8 with 25.1 and 5.5 (allow $\pm$ ) Allow use of $z = 1.04$ 2 <sup>nd</sup> M1 for $1 - p$ (where $0.84 < p < 0.86$ ) A1cso for an answer of 0.1492... or better (calc: 0.1500...) with evidence of both M's seen	
(b)	M1 for standardising their letter $y$ with 25.1 and 5.5 and setting equal to $z$ value $1 <  z  < 2$ B1 for use of $z = \pm 1.6449$ or better (calc 1.6448536...) with the correct standardisation. A1 for awrt 16.1 (ISW)(calc 16.053305...) or range [16.1, 30.8](Allow $30.8 - 16.1 = 14.7[5]$ ) [awrt 16.05 scores 3/3 16.1 scores M1B0A1 unless 1.6449 or better is seen]	
Ans only		
(c)(i)	1 <sup>st</sup> M1 for a correct method to calculate $P(H < d)$ implied by $z = \text{awrt } 0.13$ Allow $0.05 + \text{awrt } 0.200 + \text{awrt } 0.300 [= 0.5505]$ 2 <sup>nd</sup> M1 for a correct standardisation $= z$ where $0.125 \text{ ,, }  z  \text{ ,, } 0.13$ A1cso both method marks awarded, no errors seen <b>and</b> awrt 25.82 or awrt 25.79 or $d = \text{awrt } 0.13 \times 5.5 + 25.1 = \text{awrt } 25.8$	
ALT	Verification 2 <sup>nd</sup> M1 allow $\frac{25.8 - 25.1}{5.5} = 0.127\dots$ or 0.13 A1 for 0.55 and 0.5517 (calc 0.5506 or better) seen	
(ii)	1 <sup>st</sup> M1 correct method for $P(H < m)$ Allow $0.05 + \text{awrt } 0.200$ implied by $ z  = [0.67 - 0.68]$ 2 <sup>nd</sup> M1 for standardising $m$ with 25.1 and 5.5 and setting equal to $z$ value $(0.65 \square  z  \square 0.69)$ 3 <sup>rd</sup> M1 for standardising $m$ with 25.1 and 5.5 and setting equal to awrt $-0.67$ oe A1 for $m = \text{awrt } 21.4$ (use of $z = 0.67$ gives 21.415... and $z = 0.68$ gives 21.36...) No need for 3 <sup>rd</sup> M1 to be awarded Answer only 21.4 gets M1M1M0A1. 21.39 gets 4/4	
(d)	M1 for $2 \times "m" + 3 \times 25.8 + 3 \times 30.8 [+n]$ where $n$ is an integer $\square 0$ Allow $m$ or ft their $m$ A1 for awrt 221 (cm)	

Question Number	Scheme	Marks
<b>ALT 1</b> <b>(c)(i)</b>	e.g. $P(H > 25.8 \mid "16.1" < H < 30.8)$ or $\frac{P(25.8 < H < 30.8)}{1 - (0.15 + 0.05)}$ $= \frac{0.8508 - 0.5517}{0.8}$ (tables) or $\frac{0.299345\dots}{0.8}$ (calc) $\approx \frac{3}{8}$	M1 M1 A1cso (3)
<b>Notes</b>		
<b>(c)(i)</b>	1 <sup>st</sup> M1 for a correct conditional probability statement ft their answer to (b) i.e. their y 2 <sup>nd</sup> M1 for a ratio of probs of the form $\frac{q}{0.8}$ where $q = 0.3$ to 1sf A1 for probability of approx $\frac{3}{8}$	