

Question Number	Scheme	Marks
1(a)	$4(x-5) < 2x-9 \Rightarrow 4x-20 < 2x-9 \Rightarrow x < \dots$	M1
	$\Rightarrow x < \frac{11}{2}$	A1
		(2)
(b)	$2x^2 - 5x - 63 = 0 \Rightarrow (2x+9)(x-7) = 0 \Rightarrow x = -\frac{9}{2}, 7$	M1
	$-\frac{9}{2} \leq x \leq 7$	dM1 A1
		(3)
(c)	$-\frac{9}{2} \leq x < \frac{11}{2}$	B1
		(1)
		(6 marks)

Notes

(a)

M1: Attempts to expand the brackets or e.g. divide by 4, collect terms and proceeds to $x \square \dots$ where $\square$ is any inequality or “=”.

A1: $x < \frac{11}{2}$ oe e.g. $x < 5.5, (-\infty, 5.5)$ Apply isw once a correct answer is seen.

(b)

M1: Multiplies out the brackets, rearranges and attempts to solve a 3TQ by any **non-calculator** means. Apply the general guidance for solving a 3TQ by factorisation, formula or completing the square. If roots are just written down, this scores M0

Note that e.g. $2x^2 - 5x - 63 = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{9}{2}, 7$ scores M0

But $2x^2 - 5x - 63 = 0 \Rightarrow x = \frac{5 \pm \sqrt{5^2 - 4(-63)(2)}}{2 \times 2}$ or $\frac{5 \pm 23}{4} = -\frac{9}{2}, 7$ scores M1

dM1: Chooses the inside region for their critical values found via a non-calculator method.

Condone strict inequalities e.g. $-\frac{9}{2} < x < 7$ or e.g. $x > -\frac{9}{2}, x < 7$

Depends on the previous mark.

A1: $-\frac{9}{2} \leq x \leq 7$ Allow “ $x \geq -\frac{9}{2}$ **and** $x \leq 7$ ” or $\left[-\frac{9}{2}, 7\right]$ or $7 \geq x \geq -\frac{9}{2}$

but **not** “ $x \geq -\frac{9}{2}$ or $x \leq 7$ ” and not “ $x \geq -\frac{9}{2}, x \leq 7$ ”

Apply isw once a correct answer is seen but there must be no contradictory inequalities.

(c)

B1: $-\frac{9}{2} \leq x < \frac{11}{2}$ Allow “ $x \geq -\frac{9}{2}$ **and** $x < \frac{11}{2}$ ” or $\left[-\frac{9}{2}, \frac{11}{2}\right)$ or $\frac{11}{2} > x \geq -\frac{9}{2}$

but **not** “ $x \geq -\frac{9}{2}$ or $x < \frac{11}{2}$ ” and not “ $x \geq -\frac{9}{2}, x < \frac{11}{2}$ ”

Apply isw once a correct answer is seen but there must be no extra inequalities.

Question Number	Scheme	Marks
2(i)	Uses a correct law of indices on 4^{2y+1} or 32^{4y} The possibilities are endless but some more common examples are: For 4^{2y+1}: $4^{2y} \times 4$, $2^{2(2y+1)}$, $(2^2)^{2y} \times 2^2$ For 32^{4y}: $(2^5)^{4y}$, $2^{5 \times 4y}$, 2^{20y}, $\left(4^{\frac{5}{2}}\right)^{4y}$, 4^{10y}	M1
	e.g. $2^{4y+2} = \frac{2^{20y}}{2} \Rightarrow 2^{4y+3} = 2^{20y} \Rightarrow 4y+3 = 20y \Rightarrow y = \dots$ or e.g. $4^{2y+1} = \frac{4^{10y}}{2} \Rightarrow 4^{2y+1} = 4^{10y - \frac{1}{2}} \Rightarrow 2y+1 = 10y - \frac{1}{2} \Rightarrow y = \dots$	dM1
	$y = \frac{3}{16}$	A1
		(3)
(i) Notes This part of the question can be solved in many different ways. If you are unsure if a particular approach deserves credit, seek advice from your TL M1: This is effectively a B mark and is for applying a correct law of indices on 4^{2y+1} or 32^{4y}. This may be seen in isolation i.e. not necessarily within the equation but not just e.g. $4 = 2^2$. See scheme for some examples. dM1: Reaches a value for y condoning arithmetic slips but with no incorrect index work e.g. $2 \times 4^{2y} = 8^{2y} \text{ or e.g. } \frac{4^{10y}}{2} = 2^{10y} \text{ or e.g. } \frac{32^{4y}}{2} = 16^{4y}$ See main scheme for examples of acceptable work. See also supplementary document for some example responses with marks. Depends on the first method mark. A1: Fully correct work leading to $y = \frac{3}{16}$ or e.g. $y = \frac{6}{32}$ or 0.1875 following award of both Method marks. Correct answers with no working in (i) score no marks.		

See extra document for some examples.

(ii)	$x\sqrt{27} + 21 = \frac{6x}{\sqrt{3}}$	
	E.g. $\sqrt{27} = 3\sqrt{3}$, $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$ or $2\sqrt{3}$, $\sqrt{27} \times \sqrt{3} = 9$	B1
	$(3\sqrt{3} - 2\sqrt{3})x = -21$ or e.g. $3x = -21\sqrt{3}$	M1
	$x = -\frac{21}{\sqrt{3}} = -7\sqrt{3}$	dM1 A1
		(4)
		(7 marks)
(ii) Notes B1: Correct surd work seen. E.g. $\sqrt{27} = 3\sqrt{3}$, $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$ or $2\sqrt{3}$, $\sqrt{27} \times \sqrt{3} = 9$ M1: Attempts to collect terms in x dM1: Attempts to write the answer in the form $a\sqrt{b}$ where $\sqrt{b}$ is irrational. Depends on the previous method mark. A1: cso $x = -7\sqrt{3}$ Correct answers with no working in (ii) score no marks.		

Question Number	Scheme	Marks
3(a)	$f(x) = \frac{16 + 24\sqrt{x} + 9x}{x}$	
	$= 16x^{-1} + 24x^{-\frac{1}{2}} + 9$	M1 A1
	$(f'(x) =) -16x^{-2} - 12x^{-\frac{3}{2}}$	M1 A1 A1cso
		(5)

(a) Notes

M1: Expands the numerator into a **3 or 4 term** quadratic expression in $\sqrt{x}$ **and** attempts to divide each term by x .

Award this mark for $(4 + 3\sqrt{x})^2 \rightarrow p + q\sqrt{x} + rx$ (or $(\sqrt{x})^2$), $p, q, r \neq 0$ followed by at least one of,

$\frac{p}{x}$ or $\frac{q}{\sqrt{x}}$ oe or r .

A1: Correct expression e.g. $16x^{-1} + 24x^{-\frac{1}{2}} + 9$ or equivalent e.g. $\frac{16}{x}$ for $16x^{-1}$ or $\frac{12}{\sqrt{x}} + \frac{12}{\sqrt{x}}$ for $24x^{-\frac{1}{2}}$

M1: Reduces a **correct index of x** by 1 when differentiating.

It is for one of $\dots x^{-1} \rightarrow \dots x^{-2}$ or $\dots x^{\frac{1}{2}} \rightarrow \dots x^{-\frac{1}{2}}$ or $\dots x^{-1} \rightarrow \dots x^{-1-1}$ or $\dots x^{\frac{1}{2}} \rightarrow \dots x^{-\frac{1}{2}-1}$

A1: One correct simplified term.

It is for $-16x^{-2}$ oe e.g. $-\frac{16}{x^2}$ or for $-12x^{-\frac{3}{2}}$ oe e.g. $-\frac{12}{x^{\frac{3}{2}}}$

Apply isw once a correct simplified term is seen.

A1cso: Fully correct and simplified expression $-16x^{-2} - 12x^{-\frac{3}{2}}$ oe e.g. $-\frac{16}{x^2} - \frac{12}{x^{\frac{3}{2}}}$

Do not allow e.g. $-\frac{16}{x^2} + -\frac{12}{x^{\frac{3}{2}}}$ and do not allow $-\frac{16}{x^2} - \frac{12}{x^{\frac{3}{2}}} + c$

Apply isw once a correct simplified expression is seen. " $f'(x) =$ " is not required.

See next page for attempts to use the product or quotient rule.

Special case:

It is possible to obtain a correct derivative with an incorrect expansion in the numerator e.g.

$$f(x) = \frac{16 + 24\sqrt{x} + kx}{x} = \frac{16}{x} + 24x^{-\frac{1}{2}} + k \Rightarrow (f'(x) =) -16x^{-2} - 12x^{-\frac{3}{2}}$$

If k is incorrect, this scores SC M1A0M1A1A0 in (a) but allow full recovery in (b)

(b)	$f'(4) = -1 - \frac{12}{8} = -2\frac{1}{2}$	M1
	$y - 25 = -\frac{5}{2}(x - 4) \Rightarrow y = -\frac{5}{2}x + 35$ or $y = -\frac{5}{2}x + c \Rightarrow 25 = -\frac{5}{2} \times 4 + c \Rightarrow c = \dots$ $\Rightarrow y = -\frac{5}{2}x + 35$	dM1 A1
		(3)
		8 marks

(b) Notes

M1: Substitutes $x = 4$ into their $f'(x)$ from part (a) to find a value which may be implied. There must have been an attempt to differentiate so do not allow for finding e.g. $f(4)$. If no substitution is seen and their value is incorrect for their derivative (you may need to check) then score M0

dM1: Attempts an equation of the tangent (**not the normal**) at (4, 25) using a correct method with the values correctly placed. If using $y = mx + c$ must reach as far as finding a value for c .

Depends on the first method mark.

A1: $y = -\frac{5}{2}x + 35$ or e.g. $y = -2.5x + 35$

Part (a) Using quotient rule:

$$f(x) = \frac{(4 + 3\sqrt{x})^2}{x} \Rightarrow f'(x) = \frac{x \times 2(4 + 3\sqrt{x}) \times \frac{3}{2}x^{-\frac{1}{2}} - (4 + 3\sqrt{x})^2}{x^2}$$

$$\text{M1 for } f'(x) = \frac{\dots x(4 + 3\sqrt{x}) \times x^{-\frac{1}{2}} - (4 + 3\sqrt{x})^2}{x^2}$$

A1 can then be scored for fully correct differentiation

Then,

M1 for an attempt to expand, collect terms and simplify to obtain $\frac{\dots x^{\frac{1}{2}} + \dots}{x^2}$

$$\text{A1 for } \frac{-12x^{\frac{1}{2}} - 16}{x^2}$$

$$\text{A1 for } -\frac{16}{x^2} - \frac{12}{x^{\frac{3}{2}}}$$

Part (a) Using product rule:

$$f(x) = \frac{(4 + 3\sqrt{x})^2}{x} = x^{-1}(4 + 3\sqrt{x})^2 \Rightarrow f'(x) = x^{-1} \times 2(4 + 3\sqrt{x}) \times \frac{3}{2}x^{-\frac{1}{2}} + (4 + 3\sqrt{x})^2 \times -x^{-2}$$

$$\text{M1 for } f'(x) = \dots x^{-1}(4 + 3\sqrt{x}) \times x^{-\frac{1}{2}} + \dots (4 + 3\sqrt{x})^2 x^{-2}$$

A1 can then be scored for fully correct differentiation

Then,

M1 for an attempt to expand, collect terms and simplify to obtain $\dots x^{-\frac{3}{2}}$ or $\dots x^{-2}$
Then the **A** marks as in the main scheme.

Question Number	Scheme	Marks
4(a)(i)	(0, 4)	B1
(ii)	(6, 13)	B1
(a) Notes (i) B1: Allow the coordinates to be written separately as $x = \dots, y = \dots$ and allow $y = 4$ as long as there is no suggestion that x is anything other than 0. Condone missing brackets e.g. 0, 4 or unusual punctuation e.g. (0;4) or a vector e.g. $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ Do not allow coordinates the wrong way round unless the correct answer is seen previously then apply isw. Condone $\frac{8}{2}$ for 4. Do not allow $P = 4$ (ii) B1: Allow the coordinates to be written separately as $x = \dots, y = \dots$ Condone missing brackets e.g. 6, 13 or unusual punctuation e.g. (6;13) or a vector e.g. $\begin{pmatrix} 6 \\ 13 \end{pmatrix}$ Do not allow coordinates the wrong way round unless the correct answer is seen previously then apply isw. Condone $\frac{26}{2}$ for 13. Allow work that implies the correct coordinates e.g. $x = 6 \rightarrow 2y = 18 + 8 = 26 \rightarrow y = 13$		

(b)	The gradient of line $l_1 = \frac{3}{2}$ oe	B1
	Perpendicular gradient $-\frac{2}{3}$	M1
	$y - 13 = -\frac{2}{3}(x - 6)$ or $y = -\frac{2}{3}x + c \Rightarrow 13 = -\frac{2}{3} \times 6 + c \Rightarrow c = \dots$	M1
	$2x + 3y - 51 = 0$	A1
		(4)
(c)	$y = 0 \Rightarrow 2x - 51 = 0 \Rightarrow x = \dots \left(\frac{51}{2} \right)$	M1
	Full and correct method of finding required area e.g. trapezium + triangle: $\frac{6}{2}("4" + "13") + \frac{1}{2} \times "13" \times \left(" \frac{51}{2} " - 6 \right)$	dM1
	$= 177.75$	A1
		(3)
		(9 marks)

Notes**(b)**

B1: The gradient of line $l_1 = \frac{3}{2}$ oe which may be implied.

M1: Attempts $l_2 = -\frac{1}{\text{their gradient of } l_1}$ which may be implied.

M1: Uses $x = 6$ and their y coordinate of Q with a changed $-\frac{3}{2}$ in an attempt to find an equation for l_2 using a correct method with the values correctly placed.

If using $y = mx + c$ must reach as far as finding a value for c .

A1: $2x + 3y - 51 = 0$ or any integer multiple of this equation. (Allow terms in any order).

(c)

M1: Substitutes $y = 0$ into their equation from part (b) to find the x coordinate of R .

dM1: For a full and correct method using their values to find the area of shape $OPQR$
e.g. trapezium + triangle. (See scheme)

e.g. rectangle + triangle + triangle $"4" \times 6 + \frac{6}{2}("13" - "4") + \frac{1}{2} \times "13" \times \left(" \frac{51}{2} " - 6 \right)$

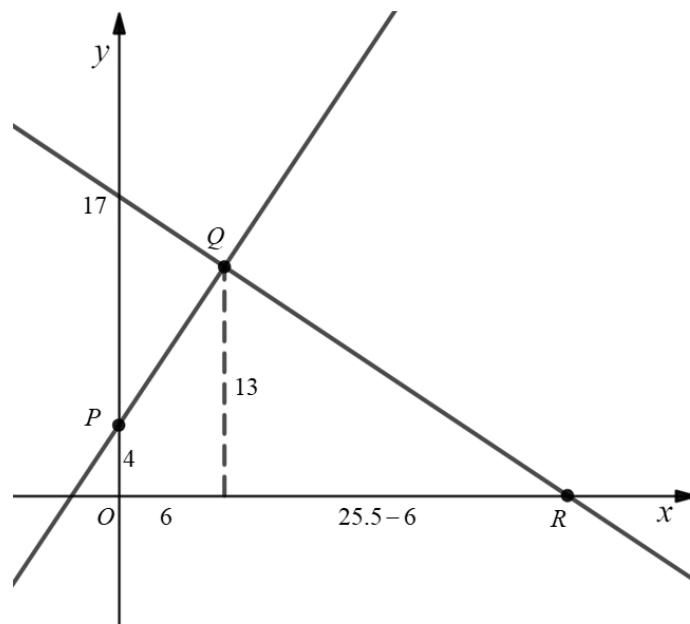
e.g. triangle PQR + triangle OPR $\frac{1}{2} \sqrt{\left(6 - " \frac{51}{2} " \right)^2 + "13"^2} \times \sqrt{6^2 + ("13" - "4")^2} + \frac{1}{2} \times " \frac{51}{2} " \times "4"$

e.g. rectangle + triangle - triangle $6 \times "13" - \frac{1}{2}("13" - "4") \times 6 + \frac{1}{2} \times "13" \times \left(" \frac{51}{2} " - 6 \right)$

May see shoelace method for areas e.g. for PQR : $\frac{1}{2} \begin{vmatrix} 0 & \frac{51}{2} & 6 & 0 \\ 2 & 4 & 0 & 13 & 4 \end{vmatrix} = \frac{1}{2} \left(\frac{51}{2} \times 13 + 6 \times 4 - \frac{51}{2} \times 4 \right)$

Depends on the first method mark.

A1: 177.75 oe e.g. $\frac{711}{4}$

4(c) Further guidance:

There are lots of different methods to find the total area so look carefully at their work.

e.g. triangle – triangle: $\frac{1}{2} \times 17 \times 25.5 - \frac{1}{2} (17 - 4) \times 6$

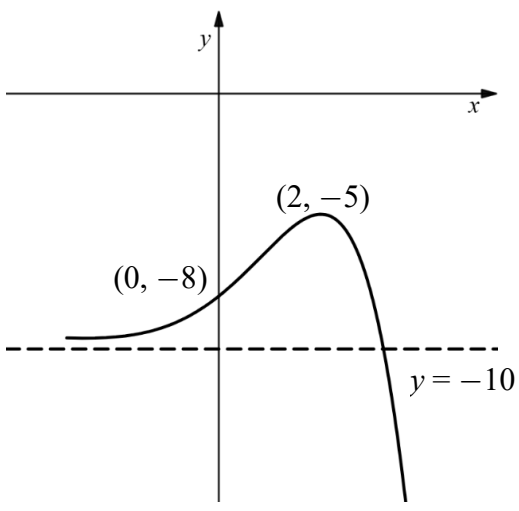
An unusual one is triangle OPQ + triangle OQR = $\frac{1}{2} \times 4 \times 6 + \frac{1}{2} \times 25.5 \times 13$

Allow integration attempts e.g. $\int_0^6 \left(\frac{3}{2}x + 4 \right) dx + \int_6^{25.5} \left(17 - \frac{2}{3}x \right) dx = \dots$

In this case, an attempt to evaluate the expression as shown is fine for the method mark and allow A1 if correctly evaluated on a calculator.

If you are unsure if a particular method deserves credit then use Review.

Question Number	Scheme	Marks
5(a)	(8, 5)	B1
		(1)
(b)	$y = 7$	B1
		(1)
(c)	$5 < k < 10$	M1A1
		(2)
Notes		
(a)		
B1: Accept (8, 5) or $x = 8, y = 5$ or a sketch of $y = f\left(\frac{1}{4}x\right)$ with a minimum point marked at (8, 5)		
Condone missing brackets e.g. 8, 5 or unusual punctuation e.g. (8; 5) or a vector e.g. $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$		
Do not allow coordinates the wrong way round unless the correct form is seen previously then apply isw.		
(b)		
B1: $y = 7$. It must be an equation and not just '7'		
(c)		
M1: For one “end” of the interval condoning strictness so allow for $k > 5, k \geq 5, k < 10, k \leq 10$		
Condone use of x or y rather than k for this mark.		
A1: $5 < k < 10$. Allow “ $k > 5$ and $k < 10$ ”, $(5, 10)$, $\{k \in \mathbb{R} : 5 < k < 10\}$		
Must be in terms of k .		
Do not allow “ $k > 5$ or $k < 10$ ” or “ $k > 5, k < 10$ ”		

(d)		Shape	B1
		(0, -8) and (2, -5)	B1
		Asymptote	B1
			(3)
			(7 marks)

(d) Notes

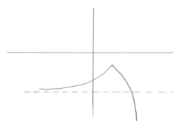
There must be a sketch to score any marks.

If the original curve is included on their sketch it can be ignored.

If more than 1 sketch is given, mark the final attempt.

- B1: For a reflection of the original curve in the x -axis.
 The curve must be in quadrants 3 and 4 only.
 There should be a maximum in quadrant 4.

Condone a maximum that is cusp-like e.g.



as long as the intention is clear.

The right hand tail should be below the left hand tail i.e. must at least touch the asymptote if it was drawn.

Condone a right hand tail that starts to bend to the right as long as it does not head back towards the x -axis to create a clear minimum.

The coordinates of the y intercept, the maximum and the position of the asymptote can be ignored for this mark.

- B1: For the intercept of (0, -8) and the maximum point (2, -5) correctly labelled that correspond with the sketch.

Allow -8 marked in the correct place and condone (-8, 0) as long as it is the correct place.

Allow the (2, -5) to be indicated by the labels 2 and -5 on the appropriate axes.

Allow to appear away from the sketch but they must be fully correct.

If there is any ambiguity, the sketch takes precedence.

- B1: For the asymptote equation of $y = -10$ that corresponds with their sketch.

It must be an **equation** i.e. not just -10 marked on the y -axis.

The curve must be asymptotic to $y = -10$ from above for one of the branches.

See extra document for some example sketches.

Question Number	Scheme	Marks
6(a)	$\frac{\sin DAO}{1.8} = \frac{\sin 0.84}{3.9} \Rightarrow DAO = \text{awrt } 0.351$ (NB $\sin DAO = 0.343681\dots$)	M1A1
		(2)
(b)	Angle $ADO = \pi - 0.84 - '0.351' = (1.95)$	M1
	Uses $AO^2 = 1.8^2 + 3.9^2 - 2 \times 1.8 \times 3.9 \cos '1.95' \Rightarrow AO = \dots$	M1
	$AO = \text{awrt } 4.86 \text{ (m)}$	A1
		(3)
	(b) Alternative 1	
	$3.9^2 = 1.8^2 + AO^2 - 2 \times 1.8 \times AO \cos 0.84$	M1
	$AO^2 - 2.41AO - 11.97 = 0 \Rightarrow AO = 4.86, \cancel{2.46}$	M1A1
	(b) Alternative 2	
	$1.8^2 = 3.9^2 + AO^2 - 2 \times 3.9 \times AO \cos '0.351'$	M1
	$AO^2 - 7.32AO + 11.97 = 0 \Rightarrow AO = 4.86, \cancel{2.46}$	M1A1
	(b) Alternative 3	
	$ON = 1.8 \cos 0.84$ or $AN = 3.9 \cos '0.351'$ (See diagram below for position of N)	M1
	$AO = 1.8 \cos 0.84 + 3.9 \cos '0.351' = \dots$	M1
	$AO = \text{awrt } 4.86 \text{ (m)}$	A1

Notes**(a)**

M1: Uses the sine rule in an attempt to find angle DAO which is not their $\sin DAO$.
The sides and angles must be in the correct position within the formula.

A1: Proceeds correctly to obtain $DAO = \text{awrt } 0.351$

Do not be concerned with incorrect labels or notation e.g. $AOD = \text{awrt } 0.351$ if the intention is clear.

For the rest of the question, allow the method marks for the use of what they think is angle DAO e.g. even if they use 0.3436...

(b)

M1: States or uses angle $ADO = \pi - 0.84 - \text{their } 0.351$ (Must be seen in (b))

May be implied by $ADO = \text{awrt } 1.95$ (rads) or by their value.

M1: Uses $AO^2 = 1.8^2 + 3.9^2 - 2 \times 1.8 \times 3.9 \cos(k\pi - 0.84 - \text{their } 0.351)$ or

$$\frac{AO}{\sin(k\pi - 0.84 - \text{their } 0.351)} = \frac{3.9}{\sin 0.84} \text{ with } k = 1 \text{ or } 2 \text{ in an attempt to find a value for } AO.$$

A1: For $AO = \text{awrt } 4.86 \text{ (m)}$ Units are not required but if any are given they must be correct.

Alternative 1:

M1: Sets up a correct quadratic equation in AO using the cosine rule with angle AOD .

M1: Collects terms, simplifies and solves a 3TQ in AO or their variable.

A1: For $AO = \text{awrt } 4.86 \text{ (m)}$ Units are not required but if any are given they must be correct.

Alternative 2:

M1: Sets up a correct quadratic equation in AO using the cosine rule with angle DAO .

M1: Collects terms, simplifies and solves a 3TQ in AO or their variable.

A1: For $AO = \text{awrt } 4.86 \text{ (m)}$ Units are not required but if any are given they must be correct.

Alternative 3:

M1: Correct attempt at either ON or AN

M1: Complete method to find a value for AO

A1: For $AO =$ awrt 4.86 (m) Units are not required but if any are given they must be correct.

Must come from correct work in part (a) unless they re-start.

(c)	$\text{Area of sector} = \frac{1}{2} \times 1.8^2 \times (2\pi - 0.84) \quad (= \text{awrt } 8.8)$ $(\text{NB } 2\pi - 0.84 = 5.44318\dots)$ or $\text{Area of triangle} = \frac{1}{2} \times 1.8 \times "4.9" \times \sin 0.84 \quad (= \text{awrt } 3.3)$ $\text{Or Area of triangle} = \frac{1}{2} \times 1.8 \times 3.9 \times \sin(\pi - 0.84 - "0.351") \quad (= \text{awrt } 3.3)$	M1
	$\text{Total Area} = \frac{1}{2} \times 1.8^2 \times (2\pi - 0.84) + \frac{1}{2} \times 1.8 \times "4.9" \times \sin 0.84 = \dots$	dM1
	$= 12.1 \text{ (m}^2\text{)}$	A1
		(3)
(d)	$\text{Arc } BCD = 1.8 \times (2\pi - 0.84) \quad (= 9.8)$	M1
	$'9.8' + 3.9 + "4.86" - 1.8 = \dots \text{ or e.g.}$ $2\pi \times 1.8 - 0.84 \times 1.8 + "4.86" - 1.8 + 3.9 = \dots$	dM1
	$\text{Perimeter of shop sign} = 16.8 \text{ (m)}$	A1
		(3)
		(11 marks)

Notes

(c)

M1: Correct attempt at the area of sector *DOBCD* **or** the area of triangle *ADO* following through their *AO* from part (b). There may be other methods seen for these areas e.g.

The area of the sector may be seen as $\pi \times 1.8^2 - \frac{1}{2} \times 1.8^2 \times 0.84$

The area of the triangle may be seen as $\frac{1}{2} \times 1.8 \times 3.9 \times \sin(\pi - 0.84 - "0.351")$ or

$\frac{1}{2} \times (1.8 \cos 0.84 + 3.9 \cos "0.351") \times 1.8 \sin 0.84$ or $= \frac{1}{2} \times 3.9 \times "4.86" \times \sin "0.351"$

dM1: Complete and correct attempt at the area of the shop sign e.g.

$$\frac{1}{2} \times 1.8^2 \times (2\pi - 0.84) + \frac{1}{2} \times 1.8 \times "4.9" \times \sin 0.84$$

Depends on the previous mark.

A1: For awrt 12.1 Units are not required but if any are given they must be correct.

Note that it is also possible via Circle + Triangle – Minor sector

e.g. $\pi \times 1.8^2 + \frac{1}{2} \times 1.8 \times "4.9" \times \sin 0.84 - \frac{1}{2} \times 1.8^2 \times 0.84$

In these cases, the scheme can be applied in the same way i.e. M1 for area of Circle – Minor sector or area of triangle and dM1 for a complete and correct method Circle + Triangle – Minor sector

Must come from correct work in part (a) unless they re-start.

(d)

M1: Attempt to find the length of the major arc using a correct method.

Accept $1.8 \times (2\pi - 0.84)$ or equivalent e.g. $2\pi \times 1.8 - 0.84 \times 1.8$

dM1: Complete and correct method for the perimeter.

A1: Awrt 16.8 Units are not required but if any are given they must be correct.

Must come from correct work in parts (a) and (b) unless they re-start.

Attempts using **degrees** are acceptable and values and example working are as follows:

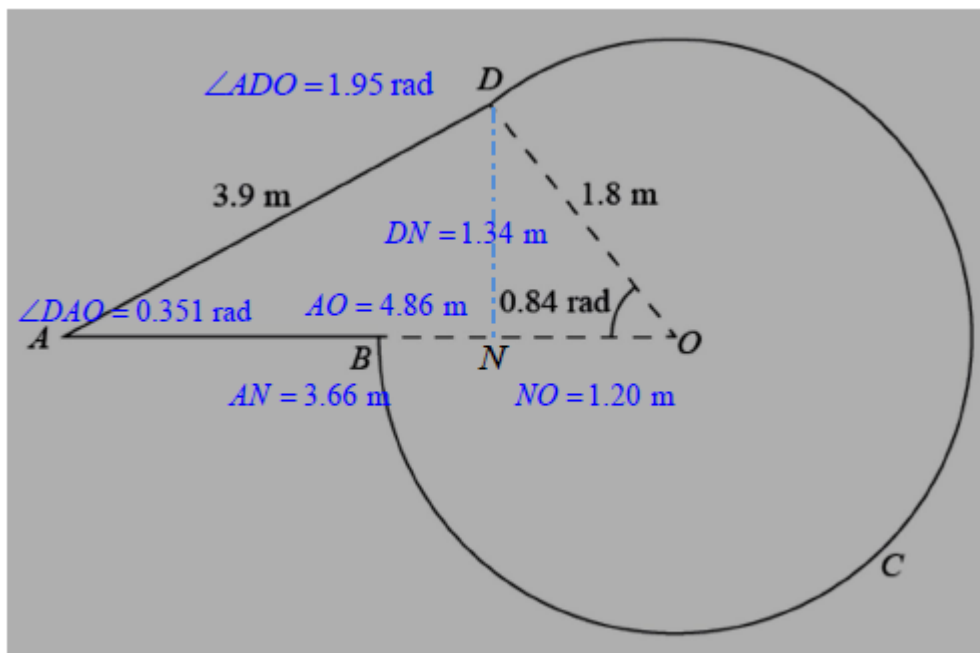
$$(a) \text{ M1: } \frac{\sin DAO}{1.8} = \frac{\sin 48}{3.9} \quad \text{A1: } DAO = 20.1^\circ = \text{awrt } 0.351$$

$$(b) \text{ M1: } ADO = \pi - 48^\circ - \text{their } 20.1^\circ \quad \text{M1: } AO^2 = 1.8^2 + 3.9^2 - 2 \times 1.8 \times 3.9 \cos 111.8^\circ$$

$$(c) \text{ M1: } \frac{360^\circ - 48^\circ}{360} \times \pi \times 1.8^2 \quad \text{or} \quad \frac{1}{2} \times 1.8 \times 4.9 \sin 48^\circ \quad \text{dM1 } \frac{360^\circ - 48^\circ}{360} \times \pi \times 1.8^2 + \frac{1}{2} \times 1.8 \times 4.9 \sin 48^\circ$$

$$(d) \text{ M1: Arc length } \frac{360^\circ - 48^\circ}{360} \times 2\pi \times 1.8$$

Diagram for reference:



Question Number	Scheme	Marks
7(a)	$(f(x)=) \int \left(3\sqrt{x} - \frac{9}{x\sqrt{x}} + \frac{4}{3} \right) dx = \int \left(3x^{\frac{1}{2}} - 9x^{-\frac{3}{2}} + \frac{4}{3} \right) dx$ $= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{4}{3}x (+c)$	M1 A1 A1
	$x = 9, y = 20 \rightarrow 20 = \frac{3(9)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9(9)^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{4}{3}(9) + c \Rightarrow c = \dots$	dM1
	$(f(x)=) 2x^{\frac{3}{2}} + 18x^{-\frac{1}{2}} + \frac{4}{3}x - 52$	A1
		(5)

(a) Notes

M1: Evidence of integration, so $x^n \rightarrow x^{n+1}$ at least once on a correct index.

It is for $3\sqrt{x} \rightarrow \dots x^{\frac{3}{2}}$ or $\frac{9}{x\sqrt{x}} \rightarrow \dots x^{-\frac{1}{2}}$ or $\frac{4}{3} \rightarrow \frac{4}{3}x$

Allow unprocessed indices e.g. $3\sqrt{x} \rightarrow \dots x^{\frac{1}{2}+1}$ or $\frac{9}{x\sqrt{x}} \rightarrow \dots x^{-\frac{3}{2}+1}$ or $\frac{4}{3} \rightarrow \frac{4}{3}x^{0+1}$

A1: One correct term which may be unsimplified but indices must be processed.

A1: Fully correct unsimplified or simplified expression.

The “+ c” is not required and $f(x) =$ is not required.

Ignore any spurious notation e.g. integral symbols.

dM1: Uses $x = 9$ and $y = 20$ in a changed expression to find their c .

Dependent upon the first method mark.

A1: $(f(x)=) 2x^{\frac{3}{2}} + 18x^{-\frac{1}{2}} + \frac{4}{3}x - 52$

Allow simplified equivalents e.g. $(f(x)=) 2x\sqrt{x} + \frac{18}{\sqrt{x}} + \frac{4}{3}x - 52$

$f(x) =$ is not required so just look for the correct expression.

Apply isw once a correct expression is seen.

If they find their constant in (b), allow the marks for that work to be scored here.

(b)	$f'(9) = 10$	B1
	Gradient of normal is $-\frac{1}{f'(9)}$	M1
	$(y - 20) = -\frac{1}{10}(x - 9)$ or $y = -\frac{1}{10}x + c \Rightarrow 20 = -\frac{1}{10} \times 9 + c \Rightarrow c = \dots$	dM1
	$x + 10y - 209 = 0$	A1
		(4)
		9 marks

(b) Notes

B1: For $f'(9) = 10$ which may be implied by the gradient of normal $= -\frac{1}{10}$

M1: For gradient of normal $= -\frac{1}{\text{their } f'(9)}$ where $f'(9)$ is an attempt to substitute $x = 9$ into the given $f'(x)$

dM1: Correct method for the equation of the normal using $x = 9, y = 20$ and a changed $f'(9)$ with the values correctly placed.

If they use $y = mx + c$ they must proceed as far as finding a value for c .

Dependent upon the previous method mark.

A1: cso $x + 10y - 209 = 0$ or any integer multiple of this equation in the required form.

Question Number	Scheme	Marks
8	$kx^2 + 8x + 2(k+7) = 0$	
	$b^2 - 4ac = 8^2 - 4k \times 2(k+7)$	M1
	$8^2 - 4k \times 2(k+7) = 64 - 8k^2 - 56k$	A1
	Attempts to solve $b^2 - 4ac \dots 0 \Rightarrow k^2 + 7k - 8 \dots 0 \Rightarrow k =$	dM1
	Critical values of 1 and -8	A1
	$k > 1$ or $k < -8$	ddM1 A1
		(6)
		6 marks

Notes

M1: Attempts $b^2 - 4ac$ with $a = k$, $b = 8$ and $c = 2(k+7)$

Condone slips with the $2(k+7)$ e.g. uses $2k+7$ or $k+7$ and condone missing brackets e.g.

$$b^2 - 4ac = 8^2 - 4k \times 2k + 14$$

May be seen embedded in an attempt to use the quadratic formula.

May also be seen in an attempt at an equation or inequality e.g. $b^2 - 4ac \dots 0$ or $b^2 < 4ac$ or e.g.

$$b^2 > 4ac$$

A1: Correct 3 term quadratic expression seen e.g. $64 - 56k - 8k^2$ which may be seen as an inequality or an equation e.g. $64 < 56k + 8k^2$ or $64 = 56k + 8k^2$ or $8 - 7k - k^2$. Allow 8^2 for 64.

Any brackets must be removed.

dM1: Attempts to find the critical values from a 3 term quadratic equation or inequality resulting from $b^2 - 4ac \dots 0$ or equivalent where ... can be any inequality or equality via **non calculator** methods.

Depends on the first method mark.

Do not allow for quoting the quadratic formula quoted followed by values e.g.

$$8k^2 + 56k - 64 = 0 \Rightarrow k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 1, -8 \text{ is M0}$$

$$\text{But allow e.g. } 8k^2 + 56k - 64 = 0 \Rightarrow k = \frac{-56 \pm \sqrt{56^2 - 4 \times 8 \times (-64)}}{2 \times 8} = 1, -8 \text{ is M1}$$

Do not allow obvious use of a calculator where the factors do not correspond with their 3TQ

$$\text{e.g. } 8k^2 + 56k - 64 = 0 \Rightarrow (k-1)(k+8) = 0 \Rightarrow k = 1, -8 \text{ is M0}$$

$$\text{but allow e.g. } 8k^2 + 56k - 64 = 0 \Rightarrow 8(k-1)(k+8) = 0 \Rightarrow k = 1, -8 \text{ is M1}$$

A1: Correct critical values. May be implied by e.g. $k^2 + 7k - 8 > 0 \Rightarrow (k-1)(k+8) > 0 \Rightarrow k > 1, k < -8$

ddM1: Chooses the outside region for their critical values. Condone e.g. $1 < k < -8$ for this mark.

Condone the boundaries being included in the inequality and condone the use of x rather than k .

The evidence for choosing the outside region must be taken from what they write down and not from e.g. a sketch.

It is dependent upon both previous Method marks.

A1: CSO $k > 1$ or $k < -8$ o.e. Must be in terms of k .

The following are acceptable:

$k > 1, k < -8$	$k > 1 \text{ and } k < -8$	$k > 1 \text{ or } k < -8$	$k > 1 \cup k < -8$
$(0, -8), (1, \infty)$	$\{k : k > 1, k < -8\}$	$k > 1 \quad k < -8$	

Question Number	Scheme	Marks
9(a)	2	B1
		(1)
(b)	1	B1
		(1)
(c)	200	B1
		(1)
(d)	0	B1
		(1)
		4 marks

Notes

You may see diagrams, calculations etc, BUT you are looking for the final answers of:

(a)

B1: 2 or equivalent e.g. “two”

(b)

B1: 1 or equivalent e.g. “one”

(c)

B1: 200 or equivalent e.g. “two hundred”

(d)

B1: 0 or equivalent e.g. “none”, “no solutions”, “no real solutions”

If more than one answer is given in any part, mark the final answer.

Question Number	Scheme	Marks
10(a)	$(f(x)=)k(x+1)(x-4)^2$	M1 A1
	Uses $(0,12) \Rightarrow 12 = 16k \Rightarrow k = \dots$	dM1 A1
	$(f(x)=)\frac{3}{4}(x+1)(x-4)^2$	
		(4)
(a) Alternative 1 by simultaneous equations		
	$(f(x)=)ax^3 + bx^2 + cx + d$ $(0, 12) \rightarrow d = 12$ $(-1, 0) \rightarrow 0 = -a + b - c + 12$ $(4, 0) \rightarrow 0 = 64a + 16b + 4c + 12$ $f'(x) = 3ax^2 + 2bx + c \rightarrow 0 = 48a + 8b + c$	M1 A1
	$a = \frac{3}{4}, b = -\frac{21}{4}, c = 6 \rightarrow (f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12$	dM1 A1
(a) Alternative 2 using roots		
	$(f(x)=)ax^3 + bx^2 + cx + d$ $(0, 12) \rightarrow d = 12$ Sum: $4 + 4 - 1 = -\frac{b}{a}$, Pair sum: $-4 - 4 + 16 = \frac{c}{a}$, product: $-16 = -\frac{12}{a}$	M1 A1
	$a = \frac{3}{4}, b = -\frac{21}{4}, c = 6 \rightarrow (f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12$	dM1 A1

(a) Notes

In part (a) “f(x)=” is not required so just look the correct expression.

M1: States or implies that $(f(x)=)k(x+1)(x+4)^2$ allowing for $k = 1$

A1: States or implies that $(f(x)=)k(x+1)(x-4)^2$ allowing for $k = 1$

dM1: Uses $(0,12)$ in $(f(x)=)k(x+1)(x+4)^2$ to find a value for k

Depends upon the first method mark.

A1: $(f(x)=)\frac{3}{4}(x+1)(x-4)^2$ oe e.g. $(f(x)=)\frac{3}{4}(x+1)(4-x)^2$

Condone a spurious “= 0” e.g. $(f(x)=)\frac{3}{4}(x+1)(x-4)^2 = 0$ Apply isw once a correct answer is seen.

Alternative 1:

M1: Starts with a general cubic and uses the given information to produce 3 equations in 3 unknowns.

A1: Correct equations including $d = 12$.

dM1: Solves to find all unknowns. You do not need to check the working as long as all values are found.

A1: $(f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12$ oe. Condone a spurious “= 0” e.g. $(f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12 = 0$

Apply isw once a correct answer is seen.

Alternative 2:

M1: Starts with a general cubic and uses the correct roots to produce 3 equations in 3 unknowns.

A1: Correct equations including $d = 12$.

dM1: Solves to find all unknowns. You do not need to check the working as long as all values are found.

A1: $(f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12$ oe. Condone a spurious “= 0” e.g. $(f(x)=)\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12 = 0$

Apply isw once a correct answer is seen.

A correct function with no working scores 4/4 in (a)

(b)	$x = -1$	B1
	$\frac{3}{4} \cancel{(x+1)} (x-4)^2 = 6 \cancel{(x+1)} (x-2) \Rightarrow x^2 - 16x + 32 = 0$	M1
	$x^2 - 16x + 32 = 0 \Rightarrow (x-8)^2 = 32 \Rightarrow x = 8 \pm 4\sqrt{2}$	dM1 A1
		(4)
(b) Alternative by expanding.		
	$x = -1$	B1
	$\frac{3}{4}x^3 - \frac{21}{4}x^2 + 6x + 12 = 6(x+1)(x-2) = 6x^2 - 6x - 12$ $\Rightarrow \frac{3}{4}x^3 - \frac{45}{4}x^2 + 12x + 24 = 0$	M1
	$\frac{3}{4}x^3 - \frac{45}{4}x^2 + 12x + 24 = 0 \Rightarrow (x+1)\left(\frac{3}{4}x^2 - 12x + 24\right) = 0$ $\frac{3}{4}x^2 - 12x + 24 = 0 \Rightarrow x = \frac{12 \pm \sqrt{12^2 - 4 \times \frac{3}{4} \times 24}}{2 \times \frac{3}{4}} = 8 \pm 4\sqrt{2}$	dM1 A1
		8 marks
(b) Notes		
(b)		
B1: States $x = -1$ Condone $(-1, 0)$ You do not need to be concerned where this comes from.		
M1: Sets their $\frac{3}{4} \cancel{(x+1)} (x-4)^2 = 6 \cancel{(x+1)} (x-2)$ cancels $(x+1)$ and proceeds to a 3TQ or factorises out $(x+1)$ to obtain a 3 term quadratic factor which must be seen and it cannot be implied by their final answer(s). It is dependent upon having achieved $f(x) = k(x+1)(x-4)^2 \quad k \neq 1$		
dM1: Dependent upon the previous method mark, it is for a non-calculator method for solving their 3TQ leading to at least one value for x .		
A1: $x = 8 \pm 4\sqrt{2}$ o.e.		
Alternative:		
B1: States $x = -1$ Condone $(-1, 0)$ You do not need to be concerned where this comes from.		
M1: Expands both sides and collects terms to one side to obtain a 4 term cubic equation and then uses the factor $(x+1)$ to find a 3 term quadratic factor which must be seen and it cannot be implied by their final answer(s). It is dependent upon having achieved $f(x) = k(x+1)(x-4)^2 \quad k \neq 1$ from the main scheme or $f(x) = ax^3 + bx^2 + cx + d, \quad a \neq 1$ from alternatives 1 and 2		
dM1: Dependent upon the previous method mark, it is for a non-calculator method for solving their 3TQ leading to at least one value for x .		
A1: $x = 8 \pm 4\sqrt{2}$ o.e.		