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| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Friday 12 January 2024

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WFM01/01**

# Mathematics

## International Advanced Subsidiary/ Advanced Level
## Further Pure Mathematics F1

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**1.**

$$\mathbf{M} = \begin{pmatrix} 2k+1 & k \\ k+7 & k+4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Show that **M** is non-singular for all real values of $k$.

**(3)**

(b) Determine $\mathbf{M}^{-1}$ in terms of $k$.

**(2)**

(a) $\det(M) = 2k^2+9k+4 - (k^2+7k)$

$= k^2+2k+4$

$= (k+1)^2+3 \geq 0$

$\det(M) \neq 0$

M IS NON-SINGULAR.

(b) $M^{-1} = \dfrac{1}{k^2+2k+4}\begin{pmatrix} k+4 & -k \\ -7-k & 2k+1 \end{pmatrix}$

**2.**

$$f(z) = 2z^3 + pz^2 + qz - 41$$

where $p$ and $q$ are integers.

The complex number $5 - 4\text{i}$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

$5+4i$

**(1)**

(b) Solve the equation $f(z) = 0$ completely.

**(4)**

(c) Determine the value of $p$ and the value of $q$.

**(2)**

When plotted on an Argand diagram, the points representing the roots of the equation $f(z) = 0$ form the vertices of a triangle.

(d) Determine the area of this triangle.

**(2)**

(b) sum = 10 prod = 25 + 16 = 41

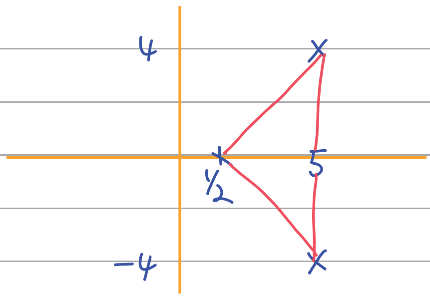
$f(z) = (z^2 - 10z + 41)(2z - 1)$

$z = 5 \pm 4i, \ \frac{1}{2}$

(c) $f(z) = 2z^3 - 21z^2 + 92z - 41$

$p = -21$ $q = 92$

(d)




AREA $= \frac{1}{2} \times 8 \times (5 - \frac{1}{2})$

$= 18$

**3.** The hyperbola $H$ has equation $xy = c^2$ where $c$ is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on $H$.

The tangent to $H$ at $P$ meets the $x$-axis at the point $A$ and meets the $y$-axis at the point $B$.

(a) Determine, in terms of $c$ and $t$,

(i) the coordinates of $A$,

(ii) the coordinates of $B$.

**(4)**

Given that the area of triangle $AOB$, where $O$ is the origin, is 90 square units,

(b) determine the value of $c$, giving your answer as a simplified surd.

**(2)**

(a) $\frac{dx}{dt} = c \quad \frac{dy}{dt} = -\frac{c}{t^2} \quad \frac{dy}{dx} = \frac{-c/t^2}{c} = -\frac{1}{t^2}$

TANGENT : $y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$

$y=0$: $ct = x - ct \quad x = 2ct \quad A(2ct, 0)$

$x=0$: $y - c/t = c/t \quad y = 2c/t \quad B(0, 2c/t)$

(b)

$\frac{1}{2} \cdot 2ct \cdot \frac{2c}{t} = 90$

$2c^2 = 90$

$c^2 = 45$

$c = 3\sqrt{5}$

**4.**

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe the single geometrical transformation represented by the matrix **A**.

STRETCH WITH SF 3 IN Y-AXIS.

**(2)**

The matrix **B** represents a rotation of 210° anticlockwise about centre (0, 0).

(b) Write down the matrix **B**, giving each element in exact form.

$B = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$

**(1)**

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Find **C**.

**(2)**

The hexagon $H$ is transformed onto the hexagon $H'$ by the matrix **C**.

(d) Given that the area of hexagon $H$ is 5 square units, determine the area of hexagon $H'$

**(2)**

(c) $C = BA = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 3 \\ -1 & -3\sqrt{3} \end{pmatrix}$

(d) $\det(C) = \frac{1}{4}\cdot 9 + \frac{1}{4}\cdot 3$

$= 3$

AREA OF $H' = 3 \times 5 = 15$

**5.** The quadratic equation

$$2x^2 - 3x + 7 = 0$$

has roots $\alpha$ and $\beta$

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$ **(1)**

$3/2$ $7/2$

(b) determine the value of $\alpha^2 + \beta^2$ **(2)**

$= (\alpha+\beta)^2 - 2\alpha\beta = 9/4 - 7 = -19/4$

(c) find a quadratic equation which has roots

$$\left(\alpha - \frac{1}{\beta^2}\right) \text{ and } \left(\beta - \frac{1}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers to be determined. **(6)**

(c) Sum $= \alpha + \beta - \left(\frac{1}{\beta^2} + \frac{1}{\alpha^2}\right)$

$= 3/2 - \dfrac{\alpha^2+\beta^2}{(\alpha\beta)^2} = 3/2 - \dfrac{-19/4}{49/4}$

$= \dfrac{185}{98}$

prod $= \alpha\beta - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + \dfrac{1}{(\alpha\beta)^2}$

$= \dfrac{7}{2} - \left(\dfrac{3/2}{7/2}\right) + \dfrac{1}{49/4}$

$= \dfrac{309}{98}$

$x^2 - \dfrac{185}{98}x + \dfrac{309}{98} = 0$

$98x^2 - 185x + 309 = 0$

**6.** (i)

$$f(x) = x - 4 - \cos\left(5\sqrt{x}\right) \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root $\alpha$ in the interval $[2.5, 3.5]$

**(2)**

(b) Use linear interpolation once on the interval $[2.5, 3.5]$ to find an approximation to $\alpha$, giving your answer to 2 decimal places.

**(2)**

(ii)

$$g(x) = \frac{1}{10}x^2 - \frac{1}{2x^2} + x - 11 \qquad x > 0$$

(a) Determine $g'(x)$.

**(2)**

The equation $g(x) = 0$ has a root $\beta$ in the interval $[6, 7]$

(b) Using $x_0 = 6$ as a first approximation to $\beta$, apply the Newton–Raphson procedure once to $g(x)$ to find a second approximation to $\beta$, giving your answer to 3 decimal places.

**(2)**

(i)
(a)

| | |
|---|---|
| 2.5 → $\frac{5}{2}$ | Ans−4−cos(5√Ans) → A: −1.448310529 |
| 3.5 → $\frac{7}{2}$ | Ans−4−cos(5√Ans) → B: 0.4975064212 |

f(x) is continuous and there is a sign change.
Hence there is a root between 2.5 and 3.5.

(b)

2.5 — α — 3.5
A — 0 — B

$$\frac{\alpha - 2.5}{0 - A} = \frac{3.5 - 2.5}{B - A}$$

$\alpha = 3.24$

$\frac{3.5-2.5}{B-A}$ → 0.5139229567
Ans×−A → 0.7443200291
Ans+2.5 → 3.244320029

**Question 6 continued**

(ii) (a) $g(x) = \frac{1}{10}x^2 - \frac{1}{2}x^{-2} + x - 11$

$g'(x) = \frac{1}{5}x + x^{-3} + 1$

(b) $x_1 = 6 - \dfrac{g(6)}{g'(6)} =$

$6 - \dfrac{A}{B}$

$6.641327173$

$6$

$6$

$\dfrac{Ans^2}{10} - .5Ans^{-2} + Ans - \triangleright$

A

$-\dfrac{509}{360}$

$6$

$6$

$.2Ans + Ans^{-3} + 1 \rightarrow B$

$\dfrac{2381}{1080}$

$x_1 = 6.641$

**7.** The parabola $C$ has equation $y^2 = \frac{4}{3}x$

The point $P\left(\frac{1}{3}t^2, \frac{2}{3}t\right)$, where $t \neq 0$, lies on $C$.

(a) Use calculus to show that the normal to $C$ at $P$ has equation

$$3tx + 3y = t^3 + 2t$$

**(3)**

The normal to $C$ at the point where $t = 9$ meets $C$ again at the point $Q$.

(b) Determine the exact coordinates of $Q$.

**(4)**

(a) $\frac{dx}{dt} = \frac{2}{3}t \quad \frac{dy}{dt} = \frac{2}{3} \quad \frac{dy}{dx} = \frac{2/3}{2/3\,t} = \frac{1}{t}$

NORMAL :

$y - \frac{2}{3}t = -t(x - \frac{1}{3}t^2)$

$3y - 2t = -3tx + t^3$

$3tx + 3y = t^3 + 2t$

(b) $t = 9$ :

$P(27, 6)$

$\begin{cases} 27x + 3y = 747 \\ y^2 = \frac{4}{3}x \end{cases} \qquad x = \frac{3}{4}y^2$

$\frac{81}{4}y^2 + 3y - 747 = 0$

$81y^2 + 12y - 2988 = 0$

$y = \frac{-12 \pm 984}{162} = 6, \ -\frac{166}{27}$

$\left(-\frac{166}{27}\right)^2 = \frac{4}{3}x \qquad x = \frac{6889}{243}$

$Q\left(\frac{6889}{243}, -\frac{166}{27}\right)$

**8.** (a) Use the standard results for summations to show that, for all positive integers $n$,

$$\sum_{r=1}^{n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n+1)^2(n-2)$$

**(4)**

(b) Hence show that, for all positive integers $n$,

$$\sum_{r=n}^{2n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n-1)(an+b)(cn+d)$$

where $a$, $b$, $c$ and $d$ are integers to be determined.

**(4)**

(a) $\sum_{r=1}^{n} 2r^3 - 3r^2 - r$

$= 2 \cdot \frac{1}{4}n^2(n+1)^2 - 3 \cdot \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(1+n)$

$= \frac{1}{2}n(n+1)\left[ n(n+1) - (2n+1) - 1 \right]$

$= \frac{1}{2}n(n+1)\left[ n^2 + n - 2n - 2 \right]$

$n^2 - n - 2$

$= \frac{1}{2}n(n+1)^2(n-2)$

(b) $2n$ : $\frac{1}{2}(2n)(2n+1)^2(2n-2)$

$-$

$n-1$ : $\frac{1}{2}(n-1)n^2(n-3)$

$= 2n(2n+1)^2(n-1) - \frac{1}{2}(n-1)n^2(n-3)$

$= \frac{1}{2}n(n-1)\left[ 4(4n^2+4n+1) - n(n-3) \right]$

$16n^2 + 16n + 4 - n^2 + 3n$

$15n^2 + 19n + 4$

$15 \quad +4$

$1 \quad +1$

$= \frac{1}{2}n(n-1)(n+1)(15n+4)$

**9.** Given that

$$\frac{3z-1}{2} = \frac{\lambda + 5\text{i}}{\lambda - 4\text{i}}$$

where $\lambda$ is a real constant,

(a) determine $z$, giving your answer in the form $x + y\text{i}$, where $x$ and $y$ are real and in terms of $\lambda$.

**(4)**

Given also that $\arg z = \dfrac{\pi}{4}$

(b) find the possible values of $\lambda$.

**(2)**

(a) $3z - 1 = \dfrac{2\lambda + 10i}{\lambda - 4i} \times \dfrac{\lambda + 4i}{\lambda + 4i}$

$= \dfrac{2\lambda^2 + 18\lambda i - 40}{\lambda^2 + 16}$

$3z = \dfrac{3\lambda^2 + 18\lambda i - 24}{\lambda^2 + 16}$

$z = \dfrac{\lambda^2 - 8}{\lambda^2 + 16} + \dfrac{6\lambda}{\lambda^2 + 16} i$

(b) $\tan\left(\dfrac{\pi}{4}\right) = 1 = \dfrac{6\lambda}{\lambda^2 - 8}$

arg (z) = π/4

$\lambda^2 - 8 = 6\lambda$

$\lambda^2 - 6\lambda - 8 = 0$

λ > 0

$\lambda = \dfrac{6 \pm \sqrt{68}}{2} = \dfrac{6 \pm 2\sqrt{17}}{2}$

$= 3 \pm \sqrt{17}$

NEED Re(z) > 0

Im(z) > 0

$\lambda = 3 + \sqrt{17}$

**10.** (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^n = 3^{n-1}\begin{pmatrix} 2n+3 & -n \\ 4n & 3-2n \end{pmatrix}$$

**(5)**

(ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 8^{2n+1} + 6^{2n-1}$$

is divisible by 7

**(5)**

(i) P(1) $n=1$ $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix} = 3^0\begin{pmatrix} 5 & -1 \\ 4 & 3-2 \end{pmatrix}$ TRUE.

ASSUME P(n) TRUE $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^n = 3^{n-1}\begin{pmatrix} 2n+3 & -n \\ 4n & 3-2n \end{pmatrix}$

NOW $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^{n+1} = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix} \cdot 3^{n-1}\begin{pmatrix} 2n+3 & -n \\ 4n & 3-2n \end{pmatrix}$

$= 3^{n-1}\begin{pmatrix} 10n+15-4n & -5n+2n-3 \\ 8n+12+4n & -4n+3-2n \end{pmatrix}$

$= 3^{n-1}\begin{pmatrix} 6n+15 & -3n-3 \\ 12n+12 & -6n+3 \end{pmatrix}$

$= 3^{(n+1)-1}\begin{pmatrix} 2n+5 & -(n+1) \\ 4n+4 & -2n+1 \end{pmatrix}$

$= 3^{(n+1)-1}\begin{pmatrix} 2(n+1)+3 & -(n+1) \\ 4(n+1) & 3-2(n+1) \end{pmatrix}$

$P(n) \rightarrow P(n+1)$ by induction.

P(1) TRUE, SO P(n) TRUE FOR $n \geq 1$

**Question 10 continued**

(ii) $P(1)$ $\quad f(1) = 8^3 + 6^1 = 7 \times 74$ $\quad$ TRUE

ASSUME $P(n)$ TRUE:

$$f(n) = 8^{2n+1} + 6^{2n-1} = 7k \qquad k \in \mathbb{Z}$$

$$\begin{aligned} f(n+1) - f(n) &= 8^{2n+3} + 6^{2n+1} - 8^{2n+1} - 6^{2n-1} \\ &= 64 \cdot 8^{2n+1} + 36 \cdot 6^{2n-1} - 8^{2n+1} - 6^{2n-1} \\ &= 63 \cdot 8^{2n+1} + 35 \cdot 6^{2n-1} \\ &= 7\left[9 \cdot 8^{2n+1} + 5 \cdot 6^{2n-1}\right] \end{aligned}$$

Since $f(n)$ is a multiple of 7,

So $f(n+1)$ is a multiple of 7.

$P(n) \rightarrow P(n+1)$ by induction.

$P(1)$ TRUE, so $P(n)$ TRUE for $n \geq 1$