Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Wednesday 21 October 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1. $$\mathrm{f}(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \qquad x > 0$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval [1.4, 1.5]

(2)

(b) Determine $\mathrm{f}'(x)$.

(3)

(c) Using $x_0 = 1.4$ as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(2)

(a)

1.4 $\frac{7}{5}$	$\mathrm{Ans}^3 - \frac{10\sqrt{\mathrm{Ans}} - 4\mathrm{Ans}}{\mathrm{Ans}^2}$ -0.4356732481
1.5 $\frac{3}{2}$	$\mathrm{Ans}^3 - \frac{10\sqrt{\mathrm{Ans}} - 4\mathrm{Ans}}{\mathrm{Ans}^2}$ 0.5983561271

$f(1.4) < 0 \qquad f(1.5) > 0$

f(x) is continuous and there is a sign change.
Hence there is a root between 1.4 and 1.5.

(b) $f(x) = x^3 - 10x^{-3/2} + 4x^{-1}$

$f'(x) = 3x^2 + 15x^{-5/2} - 4x^{-2}$

(c) $x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} =$

$1.4 - \frac{A}{B}$ 1.442268823

1.4 $\frac{7}{5}$	$\mathrm{Ans}^3 - \frac{10\sqrt{\mathrm{Ans}} - 4\mathrm{Ans}}{\mathrm{Ans}^2}$ -0.4356732481	A
1.4 $\frac{7}{5}$	$3\mathrm{Ans}^2 + 15\mathrm{Ans}^{-2.5} - 4\mathrm{A}$ 10.30720093	B

Leave blank

2. The quadratic equation

$$5x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$ **(1)**

$2/5$ $3/5$

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4/25 - 6/5 = -26/25$

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 8/125 - 3 \cdot 3/5 \cdot (2/5) = -82/125$ **(4)**

(c) determine a quadratic equation that has roots

$$(\alpha + \beta^2) \text{ and } (\beta + \alpha^2)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers. **(4)**

(c) Sum : $\alpha+\beta + \alpha^2+\beta^2 = 2/5 + \frac{-26}{25} = -16/25$

prod : $(\alpha+\beta^2)(\beta+\alpha^2) = \alpha\beta + \alpha^3 + \beta^3 + \alpha^2\beta^2$

$= 3/5 + -82/125 + 9/25$

$= \frac{38}{125}$

$x^2 + \frac{16}{25}x + \frac{38}{125} = 0$

$125x^2 + 80x + 38 = 0$

Leave blank

3.
$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are integers.

The complex numbers $3 + i$ and $-1 - 2i$ are roots of the equation $f(z) = 0$

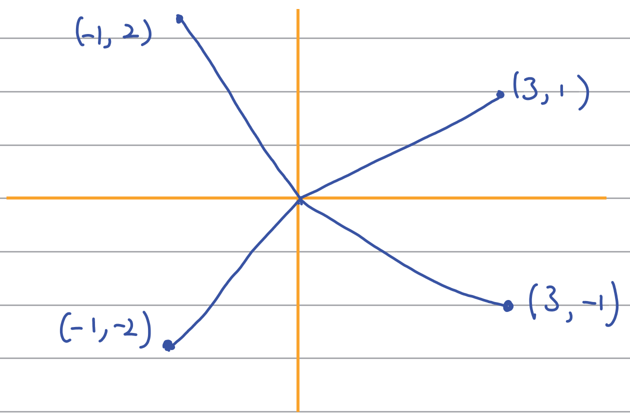
(a) Write down the other roots of this equation. **(2)**

(b) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram. **(2)**

(c) Determine the values of a, b, c and d. **(5)**

(a) $3 - i$ and $-1 + 2i$

(b)



(c) $3 \pm i \longrightarrow z^2 - 6z + 10$

$-1 \pm 2i \longrightarrow z^2 + 2z + 5$

$(z^2 - 6z + 10)(z^2 + 2z + 5)$

$a = -4$ $\quad b = 3$ $\quad c = -10$ $\quad d = 50$

Leave blank

4. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(4n^2-1)$$

for all positive integers n.

(5)

(b) Hence find the exact value of the sum of the squares of the odd numbers between 200 and 500

(4)

(a)

$$\sum_{r=1}^{n} 4r^2 - 4r + 1 = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + n$$

$$= 4 \cdot \frac{1}{6}n(n+1)(2n+1) - 4 \cdot \frac{1}{2}n(n+1) + n$$

$$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$$

$$= \frac{2}{3}n(n+1)(2n+1) - 2n^2 - n$$

$$= \frac{1}{3}n\left[2(n+1)(2n+1) - 6n - 3\right]$$

$$(2n+2)(2n+1) - 6n - 3$$

$$= \frac{1}{3}n\left[4n^2 - 1\right]$$

A−B

19499950

(b) $2r-1 = 499$, $r = 250$

$2r-1 = 201$, $r = 101$

$$\sum_{r=101}^{250}(2r-1)^2 =$$

$\sum_{x=101}^{250}((2x-1)^2)$

19499950

$\frac{250}{3}(4\times250^2-1)\rightarrow A$

20833250

$\frac{100}{3}(4\times100^2-1)\rightarrow B$

1333300

Leave blank

5. The rectangular hyperbola H has equation $xy = 64$

The point $P\left(8p, \dfrac{8}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = 8(p^4 - 1)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Determine, in terms of p, the coordinates of Q, giving your answers in simplest form.

(4)

1a) $x\dfrac{dy}{dx} + y = 0$

$\dfrac{dy}{dx} = \dfrac{-y}{x} = \dfrac{-8/p}{8p} = -\dfrac{1}{p^2}$

NORMAL : $y - 8/p = p^2(x - 8p)$

$yp - 8 = p^3x - 8p^4$

$p^3x - py = 8(p^4 - 1)$

(b) $xy = 64$

$y = 64/x$

$p^3x - p\dfrac{64}{x} = 8(p^4 - 1)$

$p^3x^2 - 64p = 8x(p^4 - 1)$

$p^3x^2 - 8x(p^4 - 1) - 64p = 0$

$x^2 \cdot p^3 + 8(1 - p^4)x - 64p = 0$

$p^3 \quad x \quad +8$

$1 \quad\quad -8p$

$(x \cdot p^3 + 8)(x - 8p) = 0$

$xp^3 = -8$

$x = -8/p^3$

$\boxed{Q\left(-8/p^3,\ -8p^3\right)}$

Leave blank

6. (i)

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.
(2)

The matrix **B** represents a rotation of 45° clockwise about the origin.

(b) Write down the matrix **B**, giving each element of the matrix in exact form.
(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Determine **C**.
(2)

(ii) The trapezium T has vertices at the points $(-2, 0)$, $(-2, k)$, $(5, 8)$ and $(5, 0)$, where k is a positive constant. Trapezium T is transformed onto the trapezium T' by the matrix

$$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$$

Given that the area of trapezium T' is 510 square units, calculate the exact value of k.
(5)

(a) STRETCH PARRALEL TO Y-AXIS
SCALE FACTOR 3
X-AXIS INVARIANT.

(b) $\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$

(c) $C = B \cdot A$

$= \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

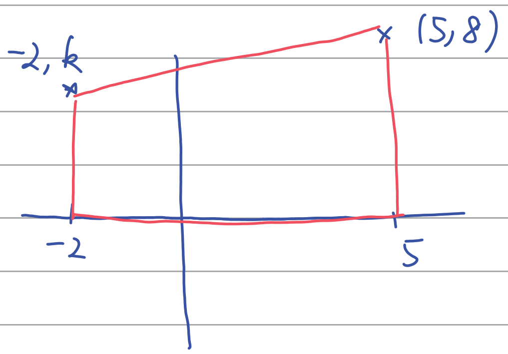
$= \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$

Leave blank

Question 6 continued

(ii) $\left|\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}\right| = |15+2| = 17$

AREA BEFORE TRANSFORMATION: $\frac{510}{17} = 30$



$\frac{1}{2} \cdot 7 \cdot (k+8) = 30$

$k+8 = 60/7$

$k = 4/7$

Leave blank

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The line l with equation $3x - 4y + 48 = 0$ is a tangent to C at the point P.

(a) Show that $a = 9$

(4)

(b) Hence determine the coordinates of P.

(2)

Given that the point S is the focus of C and that the line l crosses the directrix of C at the point A,

(c) determine the exact area of triangle PSA.

(4)

(a) $3x = 4y - 48$

$x = \frac{1}{3}(4y-48)$

$y^2 = 4a \cdot \frac{1}{3}(4y-48)$

$3y^2 = 16ay - 192a$

$3y^2 - 16ay + 192a = 0$

$\Delta = 256a^2 - 12(192)a = 0$

$256a = 2304$

$a = 9$

(b) $3y^2 - 144y + 1728 = 0$

$y^2 - 48y + 576 = 0$

$(y-24)^2 = 0$

$y = 24$

$P(16, 24)$

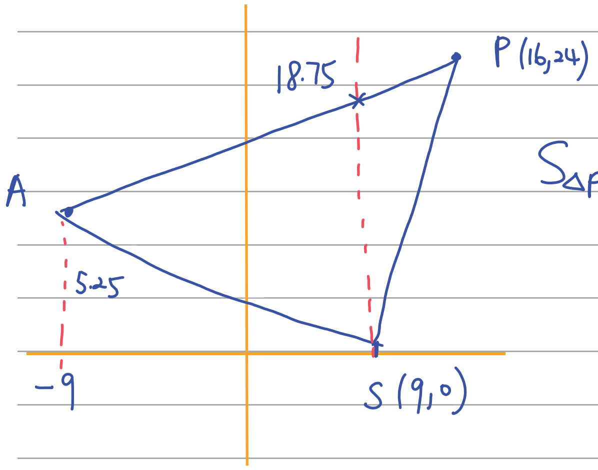
(c) $y^2 = 36 \cdot x = 4ax$ $\quad S(9,0)$

$x = -9$: l: $3(-9) - 4y + 48 = 0$

$4y = 21$ $\quad y = 5.25$

$x = 9$: l: $27 - 4y + 48 = 0$

$4y = 75$ $\quad y = 18.75$



$S_{\Delta PSA} = \frac{1}{2} \times 18 \times 18.75 + \frac{1}{2} \times 18.75 \times 7$

$= \frac{1875}{8} = 234.375$

Leave blank

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

(i) $P(1)$: $\frac{2(1)^2-1}{1^2(1+1)^2} = \frac{1}{4} = \frac{1^2}{(1+1)^2}$ TRUE

ASSUME $P(n)$: $\sum_{r=1}^{n} \frac{2r^2-1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$

NOW $\sum_{r=1}^{n+1} \frac{2r^2-1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2} + \frac{2(n+1)^2-1}{(n+1)^2(n+2)^2}$

$= \frac{n^2(n+2)^2 + 2(n^2+2n+1) - 1}{(n+1)^2(n+2)^2}$

$= \frac{n^2(n^2+4n+4) + 2n^2+4n+1}{(n+1)^2(n+2)^2}$

$= \frac{n^4+4n^3+6n^2+4n+1}{(n+1)^2(n+2)^2} = \frac{(n+1)^4}{(n+1)^2(n+2)^2}$

$= \frac{(n+1)^2}{(n+2)^2}$ $P(n) \longrightarrow P(n+1)$ by induction

Leave blank

Question 8 continued

(ii) $P(1)$: $f(1) = 12 + 2\times5^0 = 14 = 2\times7$

$P(1)$ is true.

Assum $P(n)$: $f(n) = 12^n + 2\times5^{n-1} = 7k$, $k\in\mathbb{Z}$

CONSIDER : $f(n+1) - f(n)$

$$= 12^{n+1} + 2\times5^n - 12^n - 2\times5^{n-1}$$
$$= 12\cdot12^n + 10\times5^{n-1} - 12^n - 2\times5^{n-1}$$
$$= 11\cdot12^n + 8\times5^{n-1}$$
$$= (7+4)\cdot12^n + 8\times5^{n-1}$$
$$= 7\cdot12^n + 4\cdot12^n + 8\times5^{n-1}$$
$$= 7\cdot12^n + 4\left[12^n + 2\times5^{n-1}\right]$$
$$= 7\cdot12^n + 7k$$
$$= 7l \quad \text{WHERE} \quad l = 12^n + k \in \mathbb{Z}$$

So $f(n+1)$ is also a multiple of 7.

$P(n) \rightarrow P(n+1)$ by induction

$P(1)$ is true, so $P(n)$ is true for all n.