Question Number	Scheme	Marks
1.	$r(r+3)=r^2+3r$ so $\sum_{r=1}^{n} r(r+3)=\sum_{r=1}^{n} r^2+3\sum_{r=1}^{n} r$ $=\frac{n}{6}(n+1)(2n+1)+3\frac{n}{2}(n+1)$	M1A1
	$=\frac{n}{6}(n+1)[2n+1+9]$	dM1
	$\left(=\frac{n}{6}(n+1)[2n+10]\right)=\frac{n}{3}(n+1)(n+5)$ or $a=3$ and $b=5$	A1
		(4)
		Total 4

Notes

M1: Attempts to expand and attempts to substitute **at least one <u>correct</u>** standard formula into their resulting expression.

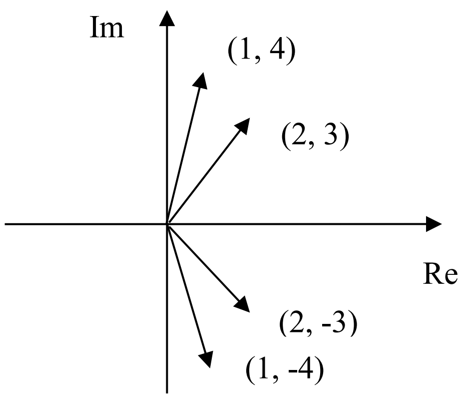
A1: Correct expression (or equivalent)

dM1: Attempt to factorise at least $n(n+1)$ **having attempted to substitute <u>both correct</u> standard formulae**. May be done in stages
Dependent on the previous M mark.

A1: Correct completion with no errors and must be in terms of n

Note: Condone work in terms of r for all but the final mark.

Attempts that use induction would generally score no marks, but if there are any cases you are unsure about please send to review

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(z)=z^4-6z^3+38z^2-94z+221$ Condone consistent work in x throughout, but mixed variables which are not recovered loses final A mark		
(a)	$z=2-3\mathrm{i}$ (is also a root)		B1
	$\left(z-(2+3\mathrm{i})\right)\left(z-(2-3\mathrm{i})\right)=\cdots$ or $(2+3\mathrm{i})+(2-3\mathrm{i})=\cdots(4)$ and $(2+3\mathrm{i})(2-3\mathrm{i})=\cdots(13)$		M1
	$z^2-4z+13$		A1
	$\dfrac{z^4-6z^3+38z^2-94z+221}{z^2-4z+13}=az^2+bz+c$ with $a\neq 0$ or $z^4-6z^3+38z^2-94z+221=(z^2-4z+13)(az^2+bz+c)\Rightarrow a=\cdots,b=\cdots,c=\cdots$		M1
	$=z^2-2z+17$		A1
	$z=\dfrac{2\pm\sqrt{4-68}}{2}=\cdots$ or $(z-1)^2+16=0\Rightarrow z=\cdots$		M1
	$=1\pm 4\mathrm{i}$		A1
			(7)
(b)		Accept points, crosses, vectors or coordinates	B1B1ft
			(2)
			Total 9

Notes

(a)

B1: $z = 2 - 3\text{i}$ seen identified as another root anywhere in the solution for (a) or (b). May only be seen at the end but must be written down somewhere in the solution. May only be on the argand diagram for (b) which is ok

M1: Correct attempt at quadratic factor by either finding the product of the two linear factors, or using the sum and product of the known roots.

A1: $z^2 - 4z + 13$ (sight of this quadratic would score first M1A1)

M1: Correct attempt at the other quadratic factor e.g. long division/ comparing coefficients/ synthetic Division/inspection using their quadratic factor to obtain a second quadratic factor. Must be some minimal working shown.

A1: $z^2 - 2z + 17$

M1: Attempt to solve their second 3TQ = 0 (we must see a method here as strict calculator warning- QF or completing the square)

A1: Correct roots.

Note: attempts that solve the quartic on a calculator with no other working score 0

(b)

If no working is shown in (a) they can score both marks in part (b) for plotting their roots correctly.

B1: $2 \pm 3\text{i}$ correctly plotted and labelled. Must be reasonably symmetric about the real axis.

B1ft: Their other conjugate pair correctly plotted and labelled. Must be reasonably symmetric about the real axis.

Note: For part (b) the axes don't need to be labelled, or could be labelled as '*x*' and '*y*' and the points can be shown with dots/crosses/vector lines/coordinates from the origin but must be clearly indicated. Accept axes labelled numerically as long as the points are identified. Condone (2, 3i) etc.

Question Number	Scheme	Marks
3.(a)	$3\left(\frac{4}{t}\right)-2(4t)=10$	M1
	$8t^2+10t-12(=0)$	A1
	$(8t-6)(t+2)=0\Rightarrow t=\cdots$ or $(4t-3)(2t+4)=0\Rightarrow t=\cdots$ or $2(4t-3)(t+2)=0\Rightarrow t=\cdots$ or $t=\frac{-10\pm\sqrt{100+384}}{16}=\cdots\left(t=\frac{3}{4},-2\right)$ or $4\left(t+\frac{5}{8}\right)^2-\frac{25}{16}-6=0\Rightarrow t=\cdots\left(t=\frac{3}{4},-2\right)$	dM1
	Points are $\left(3,\ \frac{16}{3}\right)$ and $(-8,\ -2)$	ddM1 A1
		(5)
	(a) Alternative	
	$y=\frac{10+2x}{3}$ or $x=\frac{3y-10}{2}$ and $xy=16$ $\Rightarrow x^2+5x-24(=0)$ or $3y^2-10y-32(=0)$	M1A1
	$x^2+5x-24=0\Rightarrow(x+8)(x-3)\Rightarrow x=\cdots$ $x^2+5x-24=0\Rightarrow x=\frac{-5\pm\sqrt{25+96}}{2}\Rightarrow x=\cdots$ $x^2+5x-24=0\Rightarrow\left(x+\frac{5}{2}\right)^2-\frac{121}{4}=0\Rightarrow x=\cdots$ $3y^2-10y-32=0\Rightarrow(3y-16)(y+2)=0\Rightarrow y=\cdots$ $3y^2-10y-32=0\Rightarrow y=\frac{10\pm\sqrt{100+384}}{6}\Rightarrow y=\cdots$ $3y^2-10y-32=0\Rightarrow3\left(y-\frac{5}{3}\right)^2-\frac{121}{3}=0\Rightarrow y=$	dM1
	Points are $\left(3,\ \frac{16}{3}\right)$ and $(-8,\ -2)$	ddM1 A1
(b)	$\left(\frac{3-8}{2},\frac{\frac{16}{3}-2}{2}\right)\to\left(-\frac{5}{2},\frac{5}{3}\right)$	M1 A1
		(2)
		(7 marks)

Notes

M1: Substitutes $x=4t$ and $y=\frac{4}{t}$ into the printed equation to obtain an equation in t only.

A1: Obtains a correct three term quadratic equation.

dM1: Correct method (e.g. factorising, quadratic formula or completing the square) of solving a 3TQ to find t = ...
Dependent on the previous M mark.

ddM1: Correct substitution for at least one of **their** values for t into the given parametric equations **and** obtains ***two sets*** of corresponding values for x and y. Could be implied by correct points provided dM1 scored.
Dependent on both previous M marks.

A1: Both correct points found. No need to label them as A and B if given in the order shown in the MS but if the order is reversed, they must be labelled. So '$(-8, -2)$ and $\left(3, \frac{16}{3}\right)$' scores A0 unless labelled correctly.

Must also be written as coordinates.

Alternative:

M1: Solves simultaneously $3y - 2x = 10$ with $xy = k$, $k \neq 0$ to form an equation in x or y only.

A1: Obtains a correct three term quadratic equation.

dM1: Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find $x = ...$ or $y = ...$
Dependent on the previous M mark.

ddM1: Correct substitution for at least one of **their** values **and** obtains ***two sets*** of corresponding values for x and y. Could be implied by correct points provided dM1 scored.
Dependent on both previous M marks.

A1: Both correct points found. No need to label them as A and B if given in the order shown in the MS but if the order is reversed, they must be labelled. So '$(-8, -2)$ and $\left(3, \frac{16}{3}\right)$' scores A0 unless labelled correctly.

Must be written as coordinates.

SC: If they show working to arrive at a correct quadratic in x, y, or t and then use a calculator to solve their quadratic, arriving at two correct points then award SC M1A1dM0ddM0A1

(b)

M1: Correct method for the midpoint of their points. Must be adding and dividing by 2

A1: Correct answer. Cannot leave unsimplified to score this mark, so they must combine the numerical values but accept e.g. $\left(-2.5, \frac{5}{3}\right)$ or $(-2.5, 1.66\cdots)$ or $\left(-2.5, 1.\dot{6}\right)$

Question Number	Scheme	Notes	Marks
4.	$(z-2\mathrm{i})(z^*-2\mathrm{i})=21-12\mathrm{i}$		
	$z^*=x-\mathrm{i}y$		B1
	$(x+\mathrm{i}y-2\mathrm{i})(x-\mathrm{i}y-2\mathrm{i})=....$ $=x^2-x(y+2)\mathrm{i}+x(y-2)\mathrm{i}+y^2-4$ $=x^2+y^2-4-4x\mathrm{i}$		M1
	$x^2+y^2-4=21$ and $4x=12$		M1
	$4x=12\Rightarrow x=....$		M1
	$x=3$		A1
	$y=\pm4$		A1
			(6)
			Total 6
Alternative 1			
	$(z-2\mathrm{i})(z^*-2\mathrm{i})=zz^*-2\mathrm{i}(z+z^*)-4$		M1
	$=(x+\mathrm{i}y)(x-\mathrm{i}y)-2\mathrm{i}(x+\mathrm{i}y+x-\mathrm{i}y)-4$		B1
	$=x^2+y^2-4x\mathrm{i}-4$ $x^2+y^2-4=21$ and $4x=12$		M1
	$4x=12\Rightarrow x=....$		M1
	$x=3$		A1
	$y=\pm4$		A1
Alternative 2			
	$(z-2\mathrm{i})(z^*-2\mathrm{i})=zz^*-2\mathrm{i}(z+z^*)-4$ $zz^*-2\mathrm{i}(z+z^*)-4=21-12\mathrm{i}$		M1
	$zz^*-4=21,\quad 2(z+z^*)=12$		M1
	$z^2-6z+25=0\ \left(\text{or}\ (z^*)^2-6z^*+25=0\right)$		B1
	$z^2-6z+25=0\ \left(\text{or}\ (z^*)^2-6z^*+25=0\right)$ $\Rightarrow z=\dfrac{6\pm\sqrt{36-100}}{2}\Rightarrow z=\cdots$ or $(z-3)^2+16=0\Rightarrow z=\cdots$		M1
	$x=3$		A1
	$y=\pm4$		A1

	Alternative 3	
	$(z-2\mathrm{i})(z^*-2\mathrm{i}) = zz^*-2\mathrm{i}(z+z^*)-4$ $zz^*-2\mathrm{i}(z+z^*)-4 = 21-12\mathrm{i}$	M1
	$zz^*-4=21, \quad 2(z+z^*)=12$	M1
	$x^2+y^2-4=21$ and $4x=12$	B1
	$4x=12 \Rightarrow x= ....$	M1
	$x=3$	A1
	$y=\pm 4$	A1

Notes

(a)

B1: Correct complex conjugate.

M1: Substitutes for z and their z^* and attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real **and** imaginary parts.

M1: Solves to obtain at least one value of x or y. There must be no clear use of a calculator.

A1: $x=3$

A1: $y=\pm 4$ (accept e.g. $z=3\pm 4\mathrm{i}$)

Alternative 1:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

B1: $z^*=x-\mathrm{i}y$ (may be implied).

M1: Compares real **and** imaginary parts.

M1: Solves to obtain at least one value of x or y. There must be no clear use of a calculator.

A1: $x=3$

A1: $y=\pm 4$ (accept e.g. $z=3\pm 4\mathrm{i}$)

Alternative 2:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real and imaginary parts.

B1: Correct quadratic.

M1: Solves (must see a method as strict calculator warning-see general rules on solving a 3TQ) to obtain at least one value of z or z^*

A1: $x=3$

A1: $y=\pm 4$ (accept e.g. $z=3\pm 4\mathrm{i}$)

Alternative 3:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real **and** imaginary parts.

B1: Use of $z^*=x-\mathrm{i}y$ Implied by use of $zz^*=x^2+y^2$ or $2(z+z^*)=4x$

M1: Solves to obtain at least one value of x or y. There must be no clear use of a calculator.

A1: $x=3$

A1: $y=\pm 4$ (accept e.g. $z=3\pm 4\mathrm{i}$)

Question Number	Scheme	Notes	Marks
5.	$x^2 - 2x + 3 = 0$		
(a)(i)	$\alpha + \beta = 2, \quad \alpha\beta = 3$		B1
(ii)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4 - 6 = -2$		M1A1
(iii)	$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 8 - 3\times3\times2 = -10$		M1A1
			(5)
(b)(i)	$(\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + 2(\alpha\beta)^2 + \beta^4 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$		B1*
ALT 1	$\alpha^4 + \beta^4 = \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$		B1*
ALT 2	$(\alpha+\beta)^4 = \alpha^4 + 4\alpha^3\beta + 6\alpha^2\beta^2 + 4\alpha\beta^3 + \beta^4$ $\Rightarrow \alpha^4 + \beta^4 = (\alpha+\beta)^4 - 4\alpha\beta(\alpha^2+\beta^2) - 6\alpha^2\beta^2$ $= (\alpha+\beta)^4 - 4\alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right] - 6\alpha^2\beta^2$ $= (\alpha+\beta)^4 - 4\alpha\beta(\alpha+\beta)^2 + 2\alpha^2\beta^2$ $= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 4\alpha^2\beta^2 + 2\alpha^2\beta^2$ $= (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$		B1*
(ii)	$\text{Sum} = \alpha^3 + \beta^3 - (\alpha+\beta) = -10 - 2 = -12$		B1
	$\text{product} = (\alpha^3 - \beta)(\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta = 27 - (4 - 18) + 3 = 44$		M1A1
	$\Rightarrow x^2 + 12x + 44 = 0$		M1A1
			(6)
			Total 11

Notes

(a)(i)

B1: Both $\alpha + \beta = 2, \quad \alpha\beta = 3$

(ii)

M1: Use of a correct identity for $\alpha^2 + \beta^2$ (May be implied by their work) e.g. $(\text{their sum})^2 - 2\times\text{their product}$

A1: -2 from a correct solution only.

(iii)

M1: Use of a correct identity for $\alpha^3 + \beta^3$ (May be implied by their work) e.g.
$(\text{their sum})^3 - 3\times\text{their product}\times\text{their sum}$

A1: -10 **from a correct solution only.**

(b)(i)

B1*: Correct algebraic proof starting with RHS. Must see full correct expansion of $\left(\alpha^2+\beta^2\right)^2$ before the printed answer and no incorrect work.

ALT 1 Starts with LHS but must see $\alpha^4+\beta^4=\alpha^4+\beta^4+2\alpha^2\beta^2-2\alpha^2\beta^2$ before the printed answer and no incorrect work

ALT 2 Starts with the expansion of $\left(\alpha+\beta\right)^4$ then rearranges to obtain $\alpha^4+\beta^4$, factorises and progresses to the correct printed answer with no errors

In all attempts accept $2\alpha^2\beta^2$ as an equivalent to $2\left(\alpha\beta\right)^2$

(ii)

NOTE: there are no ft marks available in part (b)

B1: Correct working without using explicit roots leading to a correct sum.
M1: Attempts to expand. Must have at least three correct terms to score this mark.
A1: Correct product.
M1: Applies $x^2-\left(\text{sum}\right)x+\text{product}$ (may have to check for their sum and product and = 0 not required for this mark)
A1: $x^2+12x+44=0$ (must have the = 0 for this mark)
If p, q, r, stated but quadratic not stated then M1A0

NOTE: any attempts which try to solve the equation to find the roots will generally score zero

Question Number	Scheme	Marks
6.(a)	$\det \mathbf{A} = 2p(5q) - 3p(3q)$	B1
	$\begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix} \to \begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$	M1
	$\mathbf{A}^{-1} = \frac{1}{pq}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$ or $\mathbf{A}^{-1} = \begin{pmatrix} \frac{5}{p} & -\frac{3}{p} \\ -\frac{3}{q} & \frac{2}{q} \end{pmatrix}$	A1
		(3)
(b)	$\mathbf{X} = \mathbf{BA}^{-1} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix} \frac{1}{pq}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix} = \cdots$ Or $\mathbf{X} = \mathbf{BA}^{-1} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}\begin{pmatrix} \frac{5}{p} & -\frac{3}{p} \\ -\frac{3}{q} & \frac{2}{q} \end{pmatrix} = \cdots$	M1
	$= \frac{1}{pq}\begin{pmatrix} 2pq & -pq \\ -3pq & 4pq \\ pq & pq \end{pmatrix}$ or equivalent e.g. $\begin{pmatrix} 5-3 & -3+2 \\ 30-33 & -18+22 \\ 25-24 & -15+16 \end{pmatrix}$ 4 correct elements from those above – simplified or unsimplified. The elements may not be in a matrix, or a matrix of the correct dimension.	A1
	$= \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	dM1A1
		(4)
	(b) Alternative	
	$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$ So $2a + 3b = 1$ *and* $3a + 5b = 1$; $a = 2, b = -1$ or $2c + 3d = 6$ *and* $3c + 5d = 11$; $c = -3, d = 4$ or $2e + 3f = 5$ *and* $3e + 5f = 8$; $e = 1, f = 1$	M1A1
	$a = 2, b = -1 \quad c = -3, d = 4 \quad e = 1, f = 1$	dM1
	$= \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	A1

Notes

(a)

B1: Correct determinant. Simplified or unsimplified.

M1: Interchanges elements in major diagonal and changes signs in other diagonal.

A1: Completely correct answer in any form, simplified or unsimplified. If the matrix is correct and they then go on to simplify incorrectly then ISW

(b)

M1: Attempts $\mathbf{BA}^{-1}$ and finds at least one element using their inverse from (a). The matrix dimension may be incorrect or not shown but one correct element for their $\mathbf{BA}^{-1}$ after an attempt at $\mathbf{BA}^{-1}$ is sufficient. The element may be unsimplified.

A1: At least 4 elements correct. The elements may not yet be simplified.

dM1: Finds a 3×2 matrix of 6 elements.

A1: Correct simplified matrix.

<u>Alternative:</u>

M1: Attempts simultaneous equations from $\mathbf{XA} = \mathbf{B}$ and solves for at least one constant.

A1: At least 4 correct values.

dM1: Finds all 6 elements for the 3×2 matrix.

A1: Correct simplified matrix.

Question Number	Scheme	Marks
7.(a)	$f(2) = 2.9497...$ or $f(2.1) = -6.0105...$ and $f(2.05) = -1.3160...$	M1A1
	$f(2.025) = 0.86846\ldots$	dM1
	Interval is (2.025, 2.05) oe	A1
		(4)
(b)	$f'(x) = -\frac{7}{2}x^{-\frac{3}{2}} - 5x^4$	M1A1
		(2)
(c)	$x_1 = 2 - \frac{f(2)}{f'(2)} = \left(2 - \frac{2.9497}{-81.237}\right) = 2.04$	M1 A1
		(2)
		Total 8

Notes

(a)

M1: Evaluates (must see numerical values) ***at least one*** of f (2) or f (2.1) **and evaluates** f (2.05)

A1: f (2) ***or*** f (2.1) correct awrt (or truncated) to 1 sf **and** f (2.05) correct awrt (or truncated) to 1 sf

dM1: Evaluates f (2.025)
Dependent on the previous M mark

A1: Needs accuracy for f (2.025) correct to 1 figure truncated or rounded and correct interval. Accept soft or hard brackets here or e.g. root/α/$x \in (2.025, 2.05)$ or root/α/$x \in [2.025, 2.05]$ or 'root belongs to (2.025, 2.05)' etc

Note:

Condone root/α/x is between 2.05 and 2.025 <u>but</u> root/α/$x \in (2.05, 2.025)$ or root/α/$x \in [2.05, 2.025]$ is **A0**

(b)

M1: For attempt at differentiation $\frac{7}{\sqrt{x}} \to ax^{-\frac{3}{2}}$ or $-x^5 \to bx^4$

A1: Correct derivative.

(c)

M1: For correct application of Newton - Raphson (using the correct formula) with $f(2)$ and $f'(2)$ which could be implied with their values.

$x_0 = 2$ followed by $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \cdots$ is M1

$x = 2 - \frac{\text{their } f(2)}{\cdots} = \cdots$ is M1

A1: cso and needs to be given to 2dp. This mark can only be scored if full marks in (b) are scored (Newton Raphson used more than once – isw)

Question Number	Scheme	Marks
8.(a)	$y^2 = 4ax$, at $P\left(ap^2, 2ap\right)$	
	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{a}\,x^{-\frac{1}{2}}$ or (implicitly) $2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4a$ or (chain rule) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2a \times \dfrac{1}{2ap}$	M1
	When $x = ap^2$, $m_T = \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{a}}{\sqrt{ap^2}} = \dfrac{\sqrt{a}}{\sqrt{a}\,p} = \dfrac{1}{p}$ or $m_T = \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4a}{2(2ap)} = \dfrac{1}{p}$	A1
	$y - 2ap = \dfrac{1}{p}\left(x - ap^2\right)$ Or $y = mx + c \Rightarrow 2ap = \dfrac{1}{p}\left(ap^2\right) + c \Rightarrow c = \cdots(ap)$	M1
	$py - 2ap^2 = x - ap^2 \Rightarrow py = x + ap^2$ Or $\Rightarrow y = \dfrac{1}{p}x + ap \Rightarrow py = x + ap^2$	A1 *
		(4)
(b)	$B\left(-a, \frac{5}{6}a\right) \Rightarrow p\left(\frac{5}{6}a\right) = -a + ap^2$ or $p\left(\frac{5}{6}a\right) = x + ap^2$ or $py = -a + ap^2$	M1
	$\Rightarrow 5ap = -6a + 6ap^2 \left(\Rightarrow 6p^2 - 5p - 6 = 0\right)$	A1
	$6p^2 - 5p - 6 = 0 \Rightarrow p = \ldots$ leading to $p = \ldots$	M1
	$\Rightarrow \left\{p = -\dfrac{2}{3} \text{ (reject)}\right\}$ $p = \dfrac{3}{2}$	A1
	$0 = x + a\left(\dfrac{3}{2}\right)^2$	M1
	$x = -\dfrac{9a}{4}$	A1
		(6)
(c)	When $p = \dfrac{3}{2}$, $y_P = 2a\left(\dfrac{3}{2}\right) = 3a$ $\text{Area}(OPD) = \dfrac{1}{2}\left(\dfrac{9a}{4}\right)(3a) = \dfrac{27a^2}{8}$ Or $\text{Area}(OPD) = \dfrac{1}{2}\begin{vmatrix} 0 & \frac{9a}{4} & -\frac{9a}{4} & 0 \\ 0 & 3a & 0 & 0 \end{vmatrix} = \dfrac{1}{2} \times 3a \times \dfrac{9a}{4}$	M1A1
		(2)
		Total 12

Notes

(a)

M1: $\frac{dy}{dx} = \pm k x^{-\frac{1}{2}}$ or $k y \frac{dy}{dx} = c$ or $\frac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$

A1: $\frac{dy}{dx} = \frac{1}{p}$

M1: Applies $y - 2ap = (\text{their } m_T)(x - ap^2)$ Where $(\text{their } m_T)$ is a function of p and has come from calculus or applies $y = mx + c$, substitutes $P(ap^2, 2ap)$ and makes progress to find the c

A1*: Obtains the printed result with at least one line of intermediate working and no incorrect work seen. E.g. Cannot go from $y - 2ap = \frac{1}{p}(x - ap^2)$ directly to the printed answer

(b)

M1: Substitutes $x = -a$ or $y = \frac{5}{6}a$ or both into **their** equation **(**or their rearranged equation**)** or the printed answer

A1: Correct equation **in any form** with $x = -a$ and $y = \frac{5}{6}a$

M1: Attempts to solve their 3TQ in p having substituted $x = -a$ and $y = \frac{5}{6}a$ (Note that this question has the less strict calculator warning, so that calculators may be used for general processing, but not to formulate a complete solution to the question, so we can allow a calculator to be used here to solve the 3TQ)

A1: $p = \frac{3}{2}$ (Can just be stated from a correct quadratic)

M1: Substitutes their "$p = \frac{3}{2}$"and $y = 0$ into **their** equation **(**or their rearranged equation**)** or the printed answer

A1: $x = -\frac{9a}{4}$

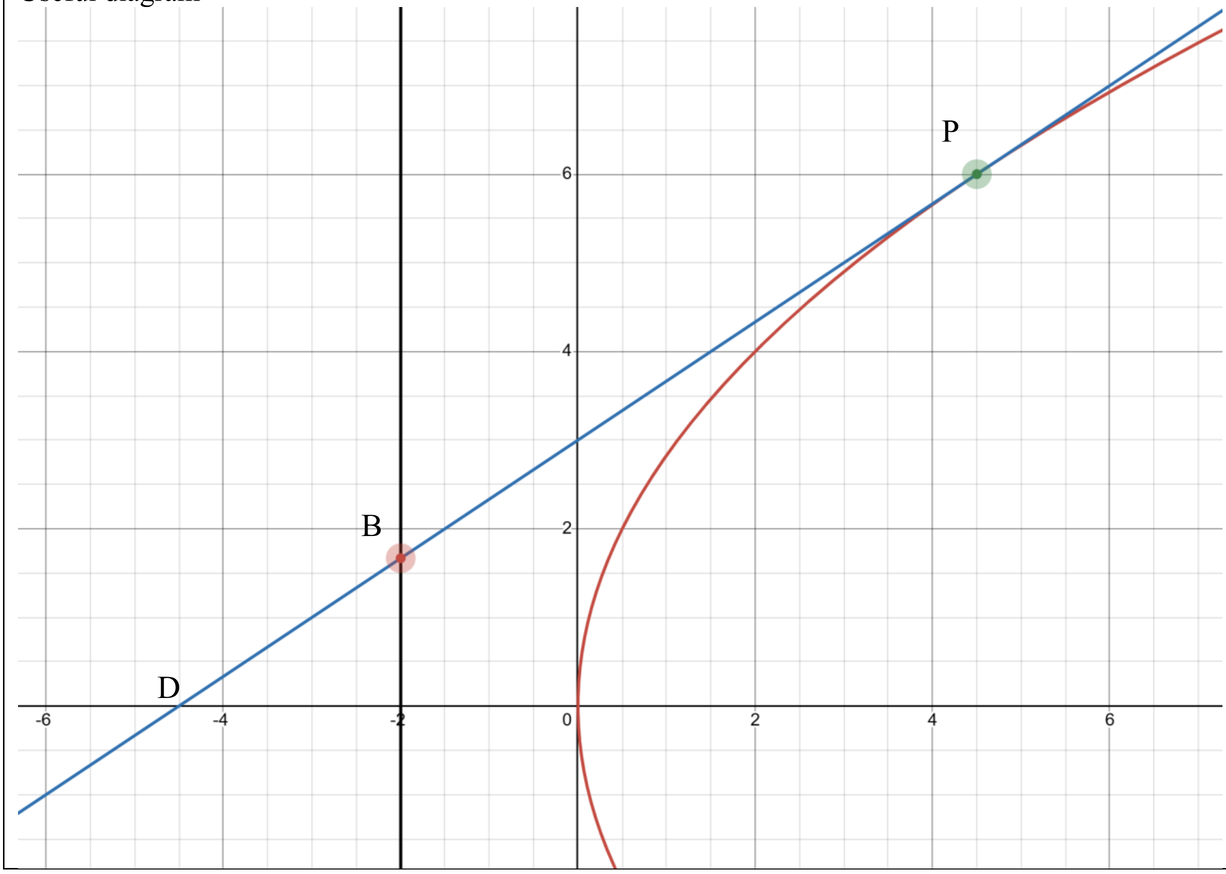
Note: If two values of p are substituted and two values for x are found, and the incorrect value of x is rejected then full marks can be scored in (b). If they don't reject the incorrect value, then the last two A marks are lost.

(c)

M1: Applies $\frac{1}{2}(\text{their } |OD|)(\text{their } y_P$ where their $y_p \neq \frac{5}{6}a)$ Allow if $OD < 0$ and a correct method in terms of a and p e.g. $\frac{1}{2} \times -ap^2 \times 2ap$

A1: $\frac{27a^2}{8}$

Useful diagram



Question Number	Scheme	Marks
9.(a)	$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$, Area$(R) = 10$, $\mathbf{B} = \mathbf{A}^4$	
	$\det(\mathbf{A}) = 6(1) - 4(1)$	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non-singular)	A1
		(2)
(b)	Area$(S) = 2(10) = 20$	M1A1
		(2)
(c)	Area$(T) = 2^4(10)$; $= 160$	M1A1
		(2)
		Total 6

Notes

(a)

M1: Correct attempt at the determinant. The working for the determinant must be seen in part (a)

A1: $\det(\mathbf{A}) = 2$ or $(6-4)$ **and** some reference to not being zero e.g. $2 \neq 0$ is sufficient

Stating e.g. det (**A**) > 0 etc is insufficient for this mark

(b)

M1: their $\det(\mathbf{A}) \times 10$ ($10 \div$ their $\det(\mathbf{A})$ is M0)

A1: 20

(c)

M1: $(\text{their } \det(\mathbf{A}))^4 \times 10$ ($10 \div (\text{their } \det(\mathbf{A}))^4$ is M0)

A1: 160

Accept use of $\mathbf{A}^2$

e.g. $\mathbf{A}^2 = \begin{pmatrix} 40 & 28 \\ 7 & 5 \end{pmatrix} \Rightarrow |\mathbf{A}^2| = 4 \Rightarrow$ Area$(T) = 4^2(10)$; $= 160$ Is acceptable

M1: $(\text{their } \det(\mathbf{A}^2))^2 \times 10$

A1: 160

BUT there must be no attempt to evaluate $\mathbf{A}^4$ to give $\det \mathbf{A}^4 = 16$

If they think $\det(\mathbf{A}) = \dfrac{1}{\det(\mathbf{A})}$ then no marks in (a) but allow M's in (b) and (c).

So e.g. if they have $\det(\mathbf{A}) = \dfrac{1}{6-4} = \dfrac{1}{2}$ in (a) then M0A0 in (a)

But if they then have $10 \times \dfrac{1}{2}$ in (b) this can score M1A0 in (b)

If they have $10 \times \left(\dfrac{1}{2}\right)^4$ in (c) then this can score M1A0 in (c)

NB $\mathbf{A}^4 = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 1796 & 1260 \\ 315 & 221 \end{pmatrix}$

Question Number	Scheme	Marks
	Condone working in other letters such as *n* etc	
10	If $n=1$, $\sum_{r=1}^{n} r^2(2r-1)=1$ and $\frac{1}{6}n(n+1)(3n^2+n-1)=1$, (LHS=RHS) (so **true for *n* = 1**).	B1
	$\sum_{r=1}^{k+1} r^2(2r-1)=\frac{1}{6}k(k+1)(3k^2+k-1)+(k+1)^2(2(k+1)-1)$	M1
	$=\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ **or** $=\frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$	A1
	$=\frac{1}{6}(k+1)(k+2)(3k^2+7k+3)=\frac{1}{6}(k+1)((k+1)+1)(3(k+1)^2+(k+1)-1)$	A1
	True for *n* = *k* + 1 if true for *n* = *k*, and as true for *n* = 1 true (by induction) **for all *n*.** (true for n = 1 may be stated at the start and not at the end which is fine)	A1cso
		(5)
		(Total 5)

Notes

The conclusion may be given as a narrative throughout the question, or it may be stated at the end, but the statements in bold are required somewhere in the solution

In all working condone the occasional missing bracket etc if recovered

B1: Shows true for $n = 1$. Don't need to see the substitution explicitly, but need to see that both sides have been evaluated when $n = 1$ and arrive at 1 for both sides- getting 1 for both sides is sufficient. Don't need to explicitly say LHS = RHS but must show clearly that both sides are equivalent by evidence of evaluating both.

M1: Adds the $(k + 1)^{th}$ term to the sum of the first k terms.

A1: Obtains $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ **or** $=\frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$ from correct work (i.e. extracts 'first' linear factor or expresses as the product of two quadratics, and allow one term to be unsimplified after the 1/6 has been extracted).

A1: Achieves this result with no errors and $3k^2+7k+3$ seen (i.e. extracts 'second' linear factor <u>and</u> progresses to showing the correct form required with $n = k + 1$). Condone '$k + 2$' in place of '$(k + 1) + 1$
Allow work that shows equivalence between
e.g. $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ and $\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$

A1: Full conclusion and all previous marks scored. As long as the conclusion is conveyed throughout the solution then this mark can be scored if all previous marks are scored.

<u>For the final mark</u>

True for $n=k \Rightarrow$ true for $n=k+1$, and as/if true for $n=1$, true for all $n\in\mathbb{N}$ (or $n\in\mathbb{Z}^+$ or integers)

Or

True for $n=k+1$ when true for $n = k$ and as/if true for $n=1$, true for all $n\in\mathbb{N}$ (or $n\in\mathbb{Z}^+$ or integers)

Condone 'true for all $n\in\mathbb{Z}$' but do not allow 'true for $n\in\mathbb{R}$' or just 'true'

Accept 'correct/it works for' in place of 'true'

Note that some candidates may resort to 'assume true for $n = k + 1$ and show true for $n = k + 2$ etc which marks the same way as above

M1: $\sum_{r=1}^{k+2} r^2(2r-1) = \frac{1}{6}(k+1)(k+2)(3(k+1)^2 + (k+1) - 1) + (k+2)^2(2(k+2)-1)$

A1: $= \frac{1}{6}(k+2)(3k^3 + 22k^2 + 52k + 39)$

A1: $= \frac{1}{6}(k+2)(k+3)(3k^2 + 13k + 13) = \frac{1}{6}(k+2)(k+3)(3(k+2)^2 + (k+2) - 1)$

A1: Full conclusion and all previous marks scored.

If there are any attempts you are not sure about, please send to review
Any attempts which do not use proof by induction score 0 marks