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1	$x^2 - 4x + c = 2x - 7 \rightarrow x^2 - 6x + c + 7 (= 0)$ $36 - 4(c + 7) < 0$ $c > 2$	M1 DM1 A1 [3]	All terms on one side Apply $b^2 - 4ac < 0$. Allow \leq .
2	$[7C2] \times \left[\left(\frac{x}{3} \right)^5 \right] \times \left[\left(\frac{9}{x^2} \right)^2 \right]$ soi $21 \times \frac{1}{3^5} (x^5) \times 81 \left(\frac{1}{x^4} \right)$ soi 7	B2,1,0 B1 B1 [4]	Seen Identified as required term Accept $7x$
3 (i)	$[3] [(x-1)^2] [-1]$	B1B1B1 [3]	
(ii)	$f'(x) = 3x^2 - 6x + 7$ $= 3(x-1)^2 + 4$ > 0 hence increasing	B1 B1 ^h DB1 [3]	Ft <i>their (i)</i> + 5 Dep B1 ^h unless other valid reason
4 (i)	Sector $OCD = \frac{1}{2}(2r)^2\theta (= 2r^2\theta)$ Sector(s) $OAB/OEF = (2)\frac{1}{2}r^2(\pi - \theta)$ Total $= r^2(\pi + \theta)$	B1 B1 B1 [3]	$2r^2\theta$ seen somewhere Accept with/without factor (2) AG www
(ii)	Arc $CD = 2r\theta$ Arc(s) $AB/EF = (2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	B1 B1 B1 B1 [4]	Accept with/without factor (2) Must be simplified

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<p>5 (i)</p>	$-2p^2 + 16p - 24 + 2p^2 - 6p + 2$ <p>Set scalar product = 0 and attempt solution $p = 2.2$</p>	<p>M1 DM1 A1 [3]</p>	<p>Good attempt at scalar product</p>
<p>(ii)</p>	$4 - 2p = 2(p - 6) \text{ or } p = 2(2p - 6)$ $p = 4 \rightarrow \vec{OA} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad \vec{OB} = \begin{pmatrix} -4 \\ 4 \\ 2 \end{pmatrix}$ $ \vec{OA} = \sqrt{(-2)^2 + 2^2 + 1^2} = 3$ <p>ALT 1 Compare AB with $OA \rightarrow 10 - 3p = p - 6$ or $6 - p = 2p - 6$. Similarly cf AB with OB</p> <p>ALT 2 $(OA \cdot OB) / (OA \times OB) = 1$ or $-1 \rightarrow$ $10p - 22 = \sqrt{5p^2 - 36p} +$ $73\sqrt{5p^2 - 16p + 20}$</p> $\rightarrow 125p^4 - 260p^3 + 941p^2 - 1448p + 976 = 0 \rightarrow p = 4$ <p>with $OA \cdot AB$ or $OB \cdot AB$.</p> <p>ALT 3 OA & OB have equal unit vectors. (Similarly with OA & AB or OB & AB.) Hence</p> $\frac{1}{\sqrt{5p^2 - 36p + 73}} \begin{pmatrix} p - 6 \\ 2p - 6 \\ 1 \end{pmatrix}$ $= \frac{1}{\sqrt{5p^2 - 16p + 20}} \begin{pmatrix} 4 - 2p \\ p \\ 2 \end{pmatrix}$ $\rightarrow \frac{1}{\sqrt{5p^2 - 36p + 73}} = \frac{2}{\sqrt{5p^2 - 16p + 20}}$ $\rightarrow 15p^2 - 128p + 272 = 0$ $\rightarrow (p - 4)(15p - 68) = 0$ $\rightarrow p = 4 \text{ (or } 68/15)$	<p>M1 A1 At least one of OA and OB correct</p> <p>M1A1 [4] For M1 accept a numerical p</p> <p>M1</p> <p>M1</p>	<p>At least one of OA and OB correct</p> <p>For M1 accept a numerical p</p>

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<p>6 (i) (a)</p> $1.92 + 1.84 + 1.76 + \dots$ $\frac{20}{2}[2 \times 1.92 + 19 \times (-0.08)]$ <p>23.2 cao</p> <p>(b)</p> $1.92 + 1.92(.96) + 1.92(.96)^2 + \dots$ $\frac{1.92(1 - .96^{20})}{1 - .96}$ <p>26.8 cao</p> <p>(ii)</p> $\frac{1.92}{1 - .96} = 48 \text{ or } \frac{0.96}{1 - 0.96} = 24 \text{ \& then}$ <p style="text-align: right;">Double AG</p>		<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1A1</p> <p>[2]</p>	<p>OR $a=0.96, d=-.04$ & ans</p> <p>doubled/adjusted</p> <p>Corr formula used with corr d & their a, n</p> <p>$a = 1, n = 21 \rightarrow 12.6$ (25.2), $a = 0.96, n = 21 \rightarrow 11.76$ (23.52)</p> <p>OR $a=.96, r=.96$ & ans /doubled/adjusted</p> <p>Corr formula used with $r=.96$ & their a, n</p> <p>$a = .96, n = 21 \rightarrow 13.82$ (27.63) $a = 1, n = 21 \rightarrow 14.39$ (28.78)</p> <p>$a = 1 \rightarrow 25$ (50) but must be doubled for M1</p> <p>$1.92 \frac{(1 - 0.96^n)}{1 - 0.96} < 48 \rightarrow 0.96^n > 0$</p> <p>(www)</p> <p>'which is true' scores SCB1</p>
<p>7 (a)</p> $1 + 3\sin^2 \theta + 4\cos \theta = 0$ $1 + 3(1 - \cos^2 \theta) + 4\cos \theta + 0$ $3\cos^2 \theta - 4\cos \theta - 4 = 0$ $\cos \theta = -2/3$ $\theta = 131.8 \text{ or } 228.2$ <p style="text-align: right;">AG</p> <p>(b)</p> $c = b/a \text{ cao}$ $d = a - b$		<p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1B1[✓]</p> <p>[6]</p> <p>B1</p> <p>B1</p> <p>[2]</p>	<p>Attempt to multiply by $\cos \theta$</p> <p>Use $c^2 + s^2 = 1$</p> <p>Ignore other solution</p> <p>Ft for $360 - 1^{\text{st}}$ soln. -1 extra solns in range</p> <p>Radians 2.30 & 3.98 scores SCB1</p> <p>Allow $D = (0, a - b)$</p>
<p>8 (i)</p> $3x + 1 \leq -1 \text{ (Accept } 3x + 1 = -1, 3a + 1 = -1)$ $x \leq -2/3 \Rightarrow \text{largest value of } a \text{ is } -2/3 \text{ (in terms of } a)$ <p>(ii)</p> $fg(x) = 3(-1 - x^2) + 1$ $fg(x) + 14 = 0 \Rightarrow 3x^2 = 12 \text{ oe (2 terms)}$ $x = -2 \text{ only}$ <p>(iii)</p> $gf(x) = -1 - (3x + 1)^2 \text{ oe}$ $gf(x) \leq -50 \Rightarrow (3x + 1)^2 \geq 49 \text{ (Allow } \leq \text{ or } =$ $3x + 1 \geq 7 \text{ or } 3x + 1 \leq -7 \text{ (one sufficient) www}$ $x \leq -8/3 \text{ only www}$		<p>M1</p> <p>A1</p> <p>[2]</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Do not allow gf in (i) to score in (iii)</p> <p>Accept $a \leq -2/3$ and $a = -2/3$</p> <p>No marks in this part for gf used</p> <p>No marks in this part for fg used</p> <p>OR attempt soln of $9x^2 + 6x - 48 + / \leq / \geq 0$</p> <p>OR $x - 2 \geq \text{ or } 3x + 8 \leq 0$ (one suffic)</p>

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<p>9 (i)</p> <p>At $x = 4$, $\frac{dy}{dx} = 2$</p> $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 2 \times 3 = 6$ <p>(ii)</p> $(y) = x + 4x^{\frac{1}{2}} (+c)$ <p>Sub $x = 4$, $y = 6 \rightarrow 6 = 4 + (4 \times 4^{\frac{1}{2}}) + c$</p> $c = -6 \rightarrow (y = x + 4x^{\frac{1}{2}} - 6$ <p>(iii)</p> <p>Eqn of tangent is $y - 6 = 2(x - 4)$ or $(6 - 0)/(4 - x) = 2$</p> <p>$B = (1, 0)$ (Allow $x = 1$) Gradient of normal = $-1/2$ $C = (16, 0)$ (Allow $x = 16$)</p> <p>Area of triangle = $\frac{1}{2} \times 15 \times 6 = 45$</p>		<p>B1</p> <p>M1A1 [3]</p> <p>B1</p> <p>M1</p> <p>A1 [3]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 [5]</p>	<p>Use of Chain rule</p> <p>Must include c</p> <p>Correct eqn thru $(4, 6)$ & with $m =$ <i>their 2</i></p> <p>[Expect eqn of normal: $y = -\frac{1}{2}x + 8$]</p> <p>Or $AB = \sqrt{45}$, $AC = \sqrt{180} \rightarrow$ Area = 45.0</p>
<p>10 (i)</p> $f'(x) = 2 - 2(x+1)^{-3}$ $f''(x) = 6(x+1)^{-4}$ <p>$f'0 = 0$ hence stationary at $x = 0$ $f''0 = 6 > 0$ hence minimum</p> <p>(ii)</p> $AB^2 = (3/2)^2 + (3/4)^2$ $AB = 1.68 \text{ or } \sqrt{45/4} \text{ oe}$ <p>(iii)</p> <p>Area under curve = $\int f(x) = x^2 - (x+1)^{-1}$</p> $= \left(1 - \frac{1}{2}\right) - \left(\frac{1}{4} - 2\right) = 9/4$ <p>(Apply limits $-\frac{1}{2} \rightarrow 1$)</p> <p>Area trap. = $\frac{1}{2}(3 + \frac{9}{4}) \times \frac{3}{2}$</p> $= 63/16 \text{ or } 3.94$ <p>Shaded area $63/16 - 9/4 + 27/16$ or 1.69</p> <p>ALT eqn AB is $y = -\frac{1}{2}x + 11/4$</p> $\text{Area} = \int -\frac{1}{2}x + 11/4 - \int 2x + (x+1)^{-2}$ $= \left[-\frac{1}{4}x^2 + \frac{11}{4}x\right] - \left[x^2 - (x+1)^{-1}\right]$ <p>Apply limits $-\frac{1}{2} \rightarrow 1$ to both integrals 27/16 or 1.69</p>		<p>B1</p> <p>B1</p> <p>B1</p> <p>B1 [4]</p> <p>M1</p> <p>A1 [2]</p> <p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1 [6]</p> <p>B1</p> <p>M1</p> <p>A1A1</p> <p>M1</p> <p>A1</p>	<p>AG</p> <p>www. Dependent on correct $f''(x)$ except $-6(x+1)^{-4} \rightarrow < 0$ MAX scores SC1</p> <p>Ignore $+c$ even if evaluated Do not penalise reversed limits</p> <p>Allow reversed subtn if final ans positive</p> <p>Attempt integration of at least one</p> <p>Ignore $+c$ even if evaluated Dep. on integration having taken place</p> <p>Allow reversed subtn if final ans positive</p>