1	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Justify the given statement	A1	[3]
2	*EITHER:* State or imply non-modular equation $2^2\left(3^x-1\right)^2=\left(3^x\right)^2$, or pair of equations $2\left(3^x-1\right)=\pm 3^x$	M1	
	Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$)	A1	
	OR: Obtain $3^x = 2$ by solving an equation or by inspection	B1	
	Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
	Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$	M1	
	Obtain final answers 0.631 and –0.369	A1	[4]
3	*EITHER:* Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$	M1*	
	Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent	A1	
	Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent	A1	
	Substitute limits x = 1 and x = 4, having integrated twice	M1(dep*)	
	Obtain answer $4(\ln 4 - 1)$, or exact equivalent	A1	
	OR1: Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u} - m\int e^{\frac{1}{2}u}\,du$	M1*	
	Obtain $2ue^{\frac{1}{2}u} - 2\int e^{\frac{1}{2}u}\,du$, or equivalent	A1	
	Integrate again and obtain $2ue^{\frac{1}{2}u} - 4e^{\frac{1}{2}u}$, or equivalent	A1	
	Substitute limits u = 0 and u = ln4, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	
	OR2: Using $u = \sqrt{x}$, or equivalent, integrate and obtain $ku\ln u - m\int u.\frac{1}{u}\,du$	M1*	
	Obtain $4u\ln u - 4\int 1\,du$, or equivalent	A1	
	Integrate again and obtain $4u\ln u - 4u$, or equivalent	A1	
	Substitute limits u = 1 and u = 2, having integrated twice or quoted $\int \ln u\,du$ as $u\ln u \pm u$	M1(dep*)	
	Obtain answer $8\ln 2 - 4$, or exact equivalent	A1	
	OR3: Integrate by parts and reach $I = \frac{x\ln x \pm x}{\sqrt{x}} + k\int \frac{x\ln x \pm x}{x\sqrt{x}}\,dx$	M1*	
	Obtain $I = \frac{x\ln x - x}{\sqrt{x}} + \frac{1}{2}I - \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx$	A1	
	Integrate and obtain $I = 2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits x = 1 and x = 4, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	[5]

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\frac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\frac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\frac{dy}{dx}$ in terms of tan t only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
		Show expression is identical to $\frac{dy}{dx}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2*A* formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector *ABC* in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above *BO* and *OC* give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle *AOB* or *AOC*, and a sector *AOB* or *AOC*.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7 **(i)** State or imply partial fractions are of the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3}$ — B1

Use a relevant method to determine a constant — M1

Obtain one of the values $A=-1$, $B=3$, $C=-1$ — A1

Obtain a second value — A1

Obtain the third value — A1 **[5]**

(ii) Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\dfrac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\dfrac{1}{3}x^2\right)^{-1}$ — M1

Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction — A1√+A1√

Multiply out fully by $Bx+C$, where $BC\neq 0$ — M1

Obtain final answer $\dfrac{1}{6}+\dfrac{5}{4}x+\dfrac{17}{72}x^2$, or equivalent — A1 **[5]**

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C*.]

[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]

8 **(a)** *EITHER:* Solve for u or for v — M1

Obtain $u=\dfrac{2\mathrm{i}-6}{1-2\mathrm{i}}$ or $v=\dfrac{5}{1-2\mathrm{i}}$, or equivalent — A1

Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent

Or: Set u or v equal to $x+\mathrm{i}y$, obtain two equations by equating real and imaginary parts and solve for x or for y — M1

OR: Using $a+\mathrm{i}b$ and $c+\mathrm{i}d$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d — M1

Obtain $b+2d=2$, $a+2c=0$, $a+d=0$ and $-b+c=3$, or equivalent — A1

Solve for one unknown — M1

Obtain final answer $u=-2-2\mathrm{i}$, or equivalent — A1

Obtain final answer $v=1+2\mathrm{i}$, or equivalent — A1 **[5]**

(b) Show a circle with centre $-\mathrm{i}$ — B1

Show a circle with radius 1 — B1

Show correct half line from 2 at an angle of $\dfrac{3}{4}\pi$ to the real axis — B1

Use a correct method for finding the least value of the modulus — M1

Obtain final answer $\dfrac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1) — A1 **[5]**

9	**(i)**	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find *d*	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	**[6]**
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	**[1]**

(iii)	*EITHER:*	Use $\frac{\overrightarrow{OA}.\overrightarrow{OD}}{\left|\overrightarrow{OD}\right|}$ to find projection ON of OA onto OD	M1	
		Obtain $ON = \frac{4}{3}$	A1	
		Use Pythagoras in triangle OAN to find AN	M1	
		Obtain the given answer	A1	
	OR1:	Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$	M1	
		Obtain answer $6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$	A1	
		Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$	M1	
		Obtain the given answer	A1	
	OR2:	Taking general point P of OD to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle OPA, or setting the derivative of $\left|\overrightarrow{AP}\right|$ to zero	M1	
		Solve and obtain $\lambda = \frac{4}{9}$	A1	
		Carry out method to calculate AP when $\lambda = \frac{4}{9}$	M1	
		Obtain the given answer	A1	
	OR3:	Use a relevant scalar product to find the cosine of AOD or ADO	M1	
		Obtain $\cos AOD = \frac{4}{9}$ or $\cos ADO = \frac{5}{3\sqrt{10}}$, or equivalent	A1	
		Use trig to find the length of the perpendicular	M1	
		Obtain the given answer	A1	
	OR4:	Use cosine formula in triangle AOD to find $\cos AOD$ or $\cos ADO$	M1	
		Obtain $\cos AOD = \frac{8}{18}$ or $\cos ADO = \frac{10}{6\sqrt{10}}$, or equivalent	A1	
		Use trig to find the length of the perpendicular	M1	
		Obtain the given answer	A1	[4]

10	(i)	State or imply $V = \pi h^3$	B1	
		State or imply $\frac{dV}{dt} = -k\sqrt{h}$	B1	
		Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$, or equivalent	M1	
		Obtain the given equation	A1	[4]
		[The M1 is only available if $\frac{dV}{dh}$ is in terms of h and has been obtained by a correct method.]		
		[Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.]		

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**